A Long Way Coming: Designing Centralized Markets with Privately Informed Buyers and Sellers*

Simon Loertscher† Leslie M. Marx‡ Tom Wilkening§

October 1, 2014

Forthcoming in *Journal of Economic Literature*

Abstract

We discuss the economics literature relevant to the design of centralized two-sided market mechanisms for environments in which both buyers and sellers have private information. The existing literature and the history of spectrum auctions, including the incentive auction currently being designed by the FCC, can be employed to analyze such mechanisms. We compare the revenue-efficiency tradeoff in an environment with private information on one side of the market versus the tradeoff with private information on both sides of the market; we provide an impossibility theorem for the efficient allocation of goods using a deficit-free mechanism when there is private information on both sides of the market; we discuss practical deficit-free mechanisms for various environments with two-sided private information; and we provide a synthesis to guide market design efforts and related research going forward.

Keywords: FCC, spectrum license auction, incentive auction, mechanism design
JEL-Classification: C72, D44, D61

*We thank Evan Kwerel, Jonathan Levy, Martha Stancill, and John Williams for their extremely valuable input. We thank seminar participants at the University of New South Wales, ESAM 2013, ESEI/UTS Center for Market Design Conference in Honor of John Ledyard, and ESEM 2014, as well as Fred Campbell, David Delacrétaz, Janet Currie, Steven Durlauf (the editor), Stephen Hanly, Tadashi Hashimoto, Paul Klemperer, Fuhito Kojima, Claudio Mezzetti, Federico Mini, David Salant, David Sappington, Ilya Segal, Steve Williams, and five anonymous referees for valuable comments on the paper. The first and third author thank the Department of Treasury and Finance of Victoria and the Australian Communications and Media Authority for helpful discussions that led to the development of the ideas for this paper. The views expressed here are those of the authors and do not necessarily reflect those of the individuals acknowledged above.

†Department of Economics, Level 4, FBE Building, University of Melbourne, 111 Barry St, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

‡The Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, USA: Email: marx@duke.edu.

§Department of Economics, Level 3, FBE Building, University of Melbourne, 111 Barry St, Victoria 3010, Australia. Email: tom.wilkening@unimelb.edu.au.

1 Introduction

Markets have long been recognized as an often astonishingly efficient means of allocating resources. Given that well-functioning markets fail to exist for a variety of valuable goods and services, it seems natural to ask whether government intervention to establish such "missing" markets would be desirable. This question is fundamental to economics, and over the past half century or so, economic debates about it have led to a rich body of research.

A number of current policy proposals concern establishing such "missing" markets through the appropriate design of centralized exchanges for buyers and sellers, who are arguably privately informed about their valuations and costs.[1] Thus, academic debates on the incentives created within such markets and their implications for designing the markets themselves have much policy relevance. In this article, we provide the historical background to current debates and review the related literature, providing a synthesis to guide market design efforts and related research going forward.

We refer to the problem of designing a centralized market where both buyers and sellers have private information as a two-sided market design problem. It is fundamentally different from the problem of designing a one-sided market, such as an auction for the sale of goods to privately informed buyers. Perhaps most strikingly, full efficiency and non-negative revenue are not necessarily compatible in two-sided markets. This contrasts with one-sided allocation problems where efficient mechanisms also generate positive revenue.

We revisit the well-known impossibility results from the literature on bilateral trade and discuss how this result extends to a variety of two-sided market settings. We then show that maximal revenue extraction has higher opportunity cost in terms of lost efficiency in a two-sided versus one-side market. Because economists designing two-sided markets will rely in part on intuition gained from one-sided markets, it is useful to be aware of such differences. We also discuss practical implementation of mechanisms that are deficit-free and explore ways in which adding additional supply into a two-sided market may improve efficiency in a two-sided market setting. We do not attempt to review every issue of concern in the design of centralized markets with privately informed buyers and sellers, but rather focus on the relationship between revenue and efficiency. This question lies at the heart of two-sided market design and is likely to be

[1] The U.S. Federal Communications Commission has proposed to develop an "incentive auction" in which spectrum licenses would be purchased from broadcasters and sold to providers of mobile wireless services. Other proposals have focused on centralized platforms for the exchange of airport landing slots, port slots, electricity, offset permits for the clearance of native vegetation, and greenhouse gas emission permits.

important to policy moving forward.

In this paper, we sometimes refer to an allocation problem in which only one side of the market is privately informed as a *primary market* allocation problem. One-sided private information is a plausible assumption when the seller (or the buyer) of the assets also chooses the mechanism. Analogously, we sometimes refer to an allocation with two-sided private information as a *secondary market* allocation problem. Two-sided private information arises when an entity other than a party to the transaction chooses the trading mechanism and organizes the exchange.

Economists such as Lerner (1944) have argued that markets offer sufficiently high value to society such that governments should actually organize markets where they do not emerge spontaneously. However, Vickrey (1961) argued forcefully against this idea, saying essentially that such counter-speculation would be excessively costly to society. Indirectly, Myerson and Satterthwaite (1983) made a similar point with their well-known impossibility theorem. Both Vickrey (1961) and Myerson and Satterthwaite (1983) show that, under mild regularity conditions, any mechanism that efficiently allocates goods between owners or producers of goods and potential buyers of the goods, each with market power and private information, will run a deficit.

In order to design a mechanism that produces an efficient allocation in these two-sided environments, the mechanism designer must be prepared to contribute resources. In the simplest example, consider one buyer and one seller of a good, each with private information about their value for or cost of the good. For efficiency, the buyer and seller must trade if and only if the buyer's value exceeds the seller's cost. Thus, an efficient mechanism requires that the buyer and seller truthfully reveal their private information. The mechanism must use the reports from the buyer and seller to determine whether trade should occur, but in order to achieve truthful revelation, the buyer's and seller's payments to and from the mechanism cannot depend upon their own reports. In the simple case in which the buyer's value and seller's cost are drawn from distributions with the same support, efficiency and truthful revelation are achieved by having the buyer pay an amount equal to the seller's reported cost, whenever the buyer's reported value exceeds that cost, and by having the seller receive a payment equal to the buyer's reported value, whenever the seller's report is less than that value. In this simple case, the mechanism designer pays a subsidy equal to the entire value of the gains from trade to the two trading parties.

The analysis of Vickrey (1961) and Myerson and Satterthwaite (1983) highlights difficulties associated with designing efficient centralized two-sided markets and has focused economists and policy makers on setups with one-sided private information (Milgrom, 2004, Chapter 1.4.1). In these one-sided settings, efficient allocations can be generated without running a deficit. Further, by getting the initial allocation right, the designer can ensure initial efficiency and mitigate the need for secondary markets, at least in the short run.[2]

While government-designed one-sided markets have long played a role in the economy, the design of auctions for the Federal Communications Commission (FCC) in 1994 to assign spectrum rights had an invigorating effect both in terms of practical design and economic research. In Figure 1 we illustrate this upsurge of interest in auctions in the economics literature by looking at JStor searches between 1980 and 2011.[3] Although 1996 is the first year in which there is a paper with "auction" in the title and "spectrum" in the text, by 1998, 50% of the papers with "auction" in the title mention "spectrum" in the text. The overall number of spectrum-related auction papers has remained large and relatively constant since this time.[4]

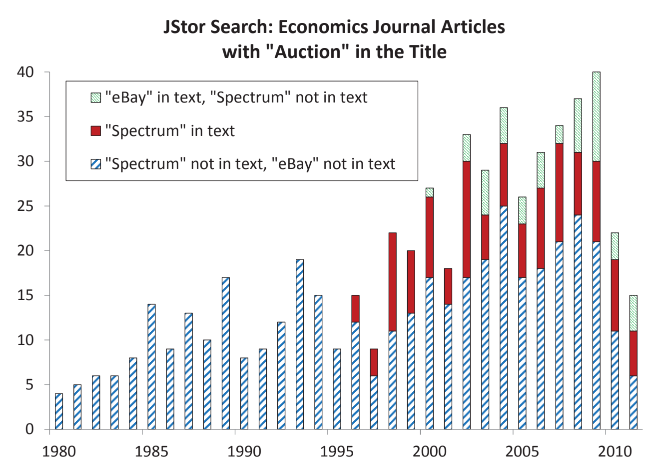

Figure 1: Growth in the economics literature on auctions as a result of the FCC spectrum auctions.

[2]This line of reasoning, which can be found, for example, in Milgrom (2004) (see also the review of Maskin (2004)), has emerged as one of the important lessons from the mechanism design literature and is now incorporated in advanced textbooks (see, e.g., Jehle and Reny (2011)).

[3]The count of papers is based on a search of JStor for articles in economics journals in English that have "auction" in the title (searching on auction* to allow for variations).

[4]As can be seen, research in internet auctions also became prevalent after 2000. A search for "auction" papers that mention eBay in the text shows papers starting in 2000, with a peak in 2009.

A recent call for the development of a centralized two-sided market reflects changes in the political/legislative environment and represents an opportunity for further economic analysis. In 2012, the U.S. Congress once again directed the FCC to design and implement a new type of mechanism, the "incentive auction," a centralized market for the exchange of spectrum licenses.[5] Based on the upsurge of academic research relating to auctions that followed the 1993 mandate to the FCC to use auctions to allocate spectrum licenses, we may now see an upsurge of interest in designing centralized two-sided markets, with more applications flowing from this research.

A key consideration in the design of one-sided auctions is that auction designs differ in the weight that they put on revenue versus efficiency.[6] This tradeoff remains central in the design of centralized two-sided markets for privately informed buyers and sellers. However, as we show, the tradeoff between revenue and efficiency is more salient in the two-sided environment because maximal revenue extraction has higher opportunity cost in terms of lost efficiency than in the one-sided environment.

Throughout the paper, we focus on environments in which utility and profit functions are quasilinear. This means that we do not review the recent matching literature that remains primarily within the domain of ordinal preferences.[7] That said, many of the key lessons that follow from the matching literature are relevant in that they suggest that large well-designed centralized two-sided markets perform better than smaller ones because size allows for more matches and that, whenever possible, efficiency should be achieved in the initial allocation because centralized secondary markets endow agents with market power.[8] However, allocating goods efficiently in the first instance is not always possible.

We begin in Section 2 by briefly reviewing the empirical context in which new types of auctions are being introduced. In Section 3, we discuss the relevant theoretical background for one-sided markets. In Section 4, we analyze the main differences between market design with one-sided versus two-sided private information. Section 5 discusses complications and

[5] An early proposal that the FCC conduct two-sided auctions of spectrum licenses was put forward by Kwerel and Williams (2002).

[6] For examples, see footnote 23.

[7] For recent reviews, see for example Sönmez and Ünver (2013) and Abdulkadiroglu and Sönmez (2013).

[8] There is a close analogy between the matching literature and the mechanism design literature reviewed here, where under fairly general conditions efficient, incentive compatible mechanisms that do not run a deficit exist for primary markets but not for centralized secondary markets: For one-to-one matching problems, it is possible to match agents efficiently via a strategy proof mechanism (the celebrated deferred acceptance algorithm of Gale and Shapley (1962)) if only agents on one side of the market have to be incentivized to reveal their preferences but not if agents on both sides have to be incentivized.

open issues. Section 6 concludes with policy implications. Topics for future research arise throughout.

2 The Context: Revenue and Efficiency

In this section we discuss how efficiency concerns and interest in revenue led to the adoption of auctions for the allocation of spectrum. We then discuss how the US and European countries chose their auction rules in light of the tradeoffs between efficiency and revenue and how revenue and efficiency can both be seen as impetus for the current development of secondary markets.

In the 1940s, Lerner (1944) advocated for government involvement in markets arguing that optimal resource allocation could be achieved in most industries through a combination of government intervention and pricing mechanisms (Lerner, 1944, p.199). In the 1950s, Herzel (1951) specifically applied this thinking to the allocation of spectrum licenses, arguing that spectrum licenses could be leased to the highest bidder via a pricing mechanism.[9] In his view, "The greatest social benefit will result if factors of production (including frequency channels) are used by producers who can pay the most for them." (Herzel, 1951, p.812) Herzel also argued that the price mechanism could guide the policies of the FCC by ensuring there was "a restraining force on the Commission of maximizing its income." Joining the debate, Coase (1959) also argued for auctioning spectrum licenses.

The arguments of Herzel and Coase were ahead of their time as their proposals faced stiff opposition. Coase (1998) describes a number of examples of such opposition, which included the argument that "the spectrum was a public good and consequently a market solution was not appropriate." (Coase, 1998, p.580)

As described by Hazlett (1998), the idea of using auctions for spectrum licenses remained shelved throughout the 1960s and 1970s; however, in the 1980s, as the demand for mobile wireless services and the number of licenses to be assigned grew, it became increasingly impractical to continue using the time-consuming administrative process used at the time. This process, known as "comparative hearings," involved an evaluation of the merits associated with different possible assignments by committees of government officials. In addition, the new mobile wireless services differed from broadcasting, which had been the dominant user, insofar as mobile

[9] Herzel explains that "Frequency channels are a socialized sector of the economy. There is a provision in the Act requiring every licensee to sign a statement that he disavows any property rights in the license issued to him." He argues that the problem and its solution are similar to "recent theoretical development in economics which use the price mechanism for the solution of the problem of how to allocate resources rationally in a socialist economy... [as] presented in Lerner, *The Economics of Control*, 1944." (Herzel, 1951, p.811)

wireless services seemed less susceptible to public interest arguments than broadcasting. This reduced the desire by key policy makers for comparative hearings. Congress declined to grant the FCC the right to auction spectrum but did allow for lotteries for non-broadcast licenses, which were authorized in 1981 and came into use in 1984.[10]

While the decision to oppose auctions in the 1950s appears to have been ideologically motivated, the decision not to authorize auctions in the 1980s appears to have been based primarily on concerns over downstream market organization.[11] A window into these concerns is provided by the questions posed to the FCC by Congress at the time.[12] The questions reveal that the major concerns centered on (1) the impact of the auction on the telecommunications industry, including the effects of auctions on incentives for innovation, warehousing,[13] and the efficient use of spectrum; (2) the effects of auctions on small and rural businesses; and (3) the concern that auctions might lead to higher prices for consumers.[14] Although a naive interpretation of the Coase Theorem may give rise to the view that initial inefficiencies from a lottery-based allocation could be remedied through subsequent trading, Coase's own advocacy for auctions proved on point when in the early 1990, the inefficiencies associated with lotteries became apparent. Lotteries were oversubscribed by individuals who were only interested in resale, and the outcome was fragmentation of license ownership across different geographic regions.[15]

A byproduct of the lottery system was the extensive trading between lottery winners and telecommunication companies via a decentralized secondary market. The large prices being paid to winners in this market may have contributed to the adoption of spectrum auctions

[10]See Hazlett (1998) for a detailed discussion of the evolution of spectrum assignment procedures. For a summary of spectrum license assignment methods used in the United States, see Hazlett (1998, Table 1), and for a timeline of unsuccessful proposals to price spectrum access from 1927 to 1992, see Hazlett (1998, Table 2).

[11]The decision not to authorize auctions for broadcast licenses may relate to concerns that if broadcasters had to pay for licenses, they might not be subject to the same public service obligations (Hazlett, 1998).

[12]Congressional Hearing on "Spectrum Auctions: FCC Proposals for the Airwaves," (U.S. House Subcommittee on Telecommunications, Consumer Protection, and Finance of the Committee on Energy and Commerce, 1 Oct. 1986. See especially the letter from Representative Timothy E. Wirth, Chairman of the Subcommittee on Telecommunications, Consumer Protection, and Finance of the Committee on Energy and Commerce, to the Chairman of the FCC.

[13]Warehousing refers to holding a spectrum license without using it to provide service to customers, perhaps to keep the license out of the hands of rivals. See Loertscher and Marx (forthcoming) for operational measures of warehousing.

[14]Other questions related to auction details and an estimate of bid preparation costs, the effect of the auction on public safety, and the interaction of the auction with spectrum already designated for allocation or in the process of being allocated.

[15]As recalled by Lawrence Krever, co-writer of the FCC's lottery order, "People tried to get more balls in the lottery drum.... Their wife applied; their parakeet applied; their dog applied."(Porter, 2013) As stated by Milgrom (2004, p.3): "The lotteries of small licenses contributed to the geographic fragmentation of the cellular industry, delaying the introduction of nationwide mobile telephone services in the United States."

because those prices highlighted to policy makers the value of spectrum (see Hazlett, 1998, Table 5). The heightened attention in the United States during the early 1990s to the state of the national budget made the prospect of auction revenues particularly attractive, and an economics literature on auctions was developing, which made clear that positive revenue could be achieved even in an efficient auction. With this backdrop, in 1993 Congress granted the FCC authority to auction licenses with multiple objectives,[16] including "efficient and intensive use of the electromagnetic spectrum" and recovering "a portion" of the value of the licenses for the public (47 U.S.C. 309(j)(3)). The FCC first began holding auctions for spectrum licenses in 1994.[17] Auctions continue to be used around the world as governments release additional spectrum for private use and technological change allows more efficient use of previously assigned spectrum.

The auction design was influenced by economics and economists. Economic theory and experimental work informed various design choices. Theory identified efficient mechanisms and optimal mechanisms for various theoretical environments.[18] Not surprisingly, theory did not identify a single optimal design for the complex real-world environment of spectrum licenses, particularly given the goals of both efficiency and recovering a portion of the value of the licenses.[19] The number of potential design variables made an exhaustive experimental study of all dimensions intractable. Instead, the general role of laboratory experiments was to test the operational rules of the auction in simple cases and to identify both unforeseen design problems and departures of strategies from those predicted by theory.[20]

[16]For a summary of the reasons that made FCC spectrum license auctions palatable to Congress in 1993, see Hazlett (1998, Table 6)

[17]The Omnibus Budget Reconciliation Act of 1993 gave the FCC authority to use auctions (Kwerel and Strack (2001)). On the performance of FCC auctions, see e.g., McMillan (1994), McAfee and McMillan (1996b), Cramton (1997), Kwerel and Rosston (2000), Marx (2006), and Brusco, Lopomo, and Marx (2009).

[18]"One plan for the auction of licenses called for a sequence of English auctions (Weber, 1993a, 1993b), a second called for a sequence of Japanese auctions (Nalebuff and Bulow, 1993a, 1993b), and a third called for simultaneous sales of licenses (McAfee, 1993a, 1993b; Milgrom and Wilson, 1993a, 1993b) . Some proposals insisted on admitting bids for bundles of geographically linked licenses, whereas others favored restricting bids to individual licenses only." (Nik-Khah, 2008, p.78)

[19]As observed by McAfee and McMillan, (1996b, p.171): "The spectrum sale is more complicated than any environment that has been studied in auction theory. No theorem exists—or can be expected to develop—that specifies the optimal auction form. The auction designers based their thinking on a range of models, each of which captures a part of the issue. The basic ideas used in designing the auction and in advising the firms on bidding strategy include the way the different bidders' values are related—they are partly idiosyncratic and partly common, or *affiliated*—and the effects of this on bidder behavior (Milgrom and Weber, 1982); how auctions reveal and aggregate dispersed information (Wilson, 1977); and the logic of bidding in the face of the winner's curse (Wilson, 1969; Milgrom and Weber, 1982). Other ideas used include the revenue increasing effect of bid discounts (Myerson, 1981; McAfee and McMillan, 1988, 1989) and reserve prices as substitutes for bidding competition (Myerson, 1981; Riley and Samuelson, 1981)."(italics in the original)

[20]Experimental evidence was influential in the question of sequential versus simultaneous bids, information revelation in the auction, concerns over cognitive limits of bidders, decisions between the simultaneous auction and sequential Japanese auctions with package bidding, withdrawal procedures, and implementation. See Plott (1997), Porter (1999), McCabe, Rassenti, and Smith (1989b), and Banks, Ledyard, and Porter (1989) for a

Results in the economics literature on the impossibility of efficient trade with two-sided private information (e.g., Myerson and Satterthwaite (1983)) provide arguments for an emphasis on efficiency in primary markets because secondary market transactions cannot be relied upon to quickly correct any inefficiencies in the initial allocation. The reality of frictions in the decentralized secondary market was evident during the period of time in which lotteries were used to allocate spectrum licenses. As argued by Milgrom (2004, p.4), "With so many parties and interests involved, the market took many years to recover from the initial fragmentation of spectrum ownership. During those years, investments were delayed and consumer services degraded. Getting the allocation right the first time does matter." However, the possibility of substituting revenue from a spectrum license auction for revenue obtained through distortionary taxes provides an argument for at least some emphasis on revenue.[21]

2.1 Revenue and Efficiency in U.S. Auctions

In resolving the tradeoff between revenue and efficiency, the FCC was attentive to the authorizing legislation for auctions (the Communications Act of 1934 as amended by the Telecom Act of 1996), which suggests that efficiency concerns should dominate revenue concerns. Section 309(j) of the Act states that one objective of the auctions is "recovery for the public of a portion of the value of the public spectrum," but it also states that "the Commission may not base a finding of public interest, convenience, and necessity solely or predominantly on the expectation of Federal revenues."

Ultimately, based on substantial input from FCC and academic economists, the FCC developed a simultaneous multiple round (SMR) auction format.[22] The SMR format, in the absence

discussion of experimental economics and its influence on early auction designs.

[21] As described by Milgrom (2004, pp.19–20), critics of an auction approach to allocating spectrum licenses have argued based on the Coase Theorem, saying: "[O]nce the licenses are issued, parties will naturally buy, sell, and swap them to correct any inefficiencies in the initial allocation. Regardless of how license rights are distributed initially, the final allocation of rights will take care of itself. Some critics have gone even farther, arguing on this basis that the only proper object of the government is to raise as much money as possible in the sale, because it should not and cannot control the final allocation." Systematic and quantitative evidence of the long-lasting effects of the inefficiencies arising from suboptimal initial allocations is provided Bleakley and Ferrie (2014) for the case of land-use rights on the Georgia frontier, which were allocated through lotteries. Bleakley and Ferrie (2014) estimate that the inefficiencies decreased the value of the land by one-fifth and persisted for as long as a century.

[22] "In a simultaneous multiple-round (SMR) auction, all licenses are available for bidding throughout the entire auction, thus the term 'simultaneous.' Unlike most auctions in which bidding is continuous, SMR auctions have discrete, successive rounds, with the length of each round announced in advance by the Commission. After each round closes, round results are processed and made public. Only then do bidders learn about the bids placed by other bidders. This provides information about the value of the licenses to all bidders and increases the likelihood that the licenses will be assigned to the bidders who value them the most. The period between auction rounds also allows bidders to take stock of, and perhaps adjust, their bidding strategies. In an SMR auction, there is no preset number of rounds. Bidding continues, round after round, until a round occurs

of reserve prices, is efficient at least in certain settings, such as when buyers have single-unit demand. With recoveries of value for the public in mind, the FCC has established minimum opening bids, but minimum opening bids and reserve prices at FCC auctions have not been aggressive.[23] This basic auction format, with various modifications and extensions, continues to be used today.[24]

2.2 Revenue and Efficiency in European Auctions

The high level of demand for spectrum relative to supply in the United States has allowed policy makers to implement efficient designs and be relatively certain of high revenue.[25] The European experience, by contrast, highlights the tradeoff between revenue and efficiency and how collusion can be a threat to both (Klemperer, 2002a).

As in the United States, revenue and efficiency both appear to have been important for the decision to use auctions to allocate radio spectrum in Europe and for the auction rules adopted by each country. For example, both the United Kingdom and Switzerland list revenue as a secondary objectives in their 3G auction design,[26] and most countries explicitly designed

in which all bidder activity ceases. That round becomes the closing round of the auction." (FCC website, http://wireless.fcc.gov/auctions/default.htm?job=about_auctions&page=2, accessed June 28, 2012)

[23] In the FCC's first auction, the minimum opening bids for the largest licenses were set at $500,000, but winning bids were $80 million. (FCC Auction 1 procedures are available at http://wireless.fcc.gov/auctions/01/releases/bip1.pdf and results are available at http://wireless.fcc.gov/auctions/01/charts/1_sum.gif.) In the FCC's 700 MHz Auction, reserve prices were based on auction results for AWS-1 spectrum licenses, which was believed to provide a conservative estimate of the market value of the licenses because the characteristics of the 700 MHz band were viewed as superior to those of the AWS-1 band. As stated in the procedures public notice for the 700 MHz Auction, "For the A, B, C, and E Blocks, we base the reserve prices on the respective market value estimates using AWS-1 bids, adding one percent, and rounding to the nearest thousand dollars. Because of the value-enhancing propagation characteristics and relatively unencumbered nature of the 700 MHz Band spectrum, we believe these are conservative estimates." (FCC Public Notice (DA 07-3415), paras. 53–54) With revenue in mind, the 700 MHz auction procedures specified that if the reserve price for the C block licenses was not met, then those licenses would be immediately reauctioned without the open access restrictions imposed on them. (See Brusco, Lopomo, and Marx (2011) for discussion of the contingent re-auction format.)

[24] A number of relatively minor modifications to the original design have been made to address susceptibility to collusion by bidders. For example, McAfee and McMillan (1996b) discuss collusive signaling and point out that the FCC could reduce such signaling by not revealing bidder identities during the auction. This design change was introduced in 2006 and implemented in 2008. A discussion of the change can be found in Marx (2006). Cramton and Schwartz (2000, 2002) find evidence of collusive conduct in spectrum auctions and discuss the effectiveness of various design modifications at suppressing collusion. Kwasnica and Sherstyuk (2001) provide experimental results that support the possibility of collusion in multiple object auctions such as those used by the FCC, particularly when the number of bidders is small. Theoretical results of Brusco and Lopomo (2002) on the possibility and profitability of collusion at multiple object ascending bid auctions are consistent with the observed empirical and experimental work.

[25] Competition between bidders is the main force by which revenue is generated in an auction. Thus, having sufficiently many interested buyers participate in an auction is often more important than getting the details of the auction exactly right. As shown by Bulow and Klemperer (1996), running an efficient auction with $N+1$ bidders will yield higher revenue for the seller than holding an optimal auction with N buyers in many situations.

[26] Barbara Roche, then Minister for Small Firms, Trade and Industry, listed the following objectives for the UK's 2000 UMTS auction: "... the Government's overall aim is to secure, for the long term benefit of UK

rules that have the potential to create inefficiency in order to increase expected revenue. The success of these rules for generating revenue has been mixed; while the UK's original UTMS auction in 2000 generated high revenues of over 600 Euros per person, subsequent auctions in the Netherlands, Italy, Switzerland, and Australia had revenues of just 170, 240, 20, and 100 Euros per person, respectively (Klemperer, 2002a).[27]

An important threat to both efficiency and revenue that became apparent during the 3G spectrum auctions in Europe was the potential for collusion among bidders. As discussed in Klemperer (2002a), both the German and Swiss auctions held in 2000 had rules that were exploited either by using bids to signal to others how to tacitly collude or through pre-auction joint bidding agreements. The experience of these early auctions has induced policy makers to set higher reserve prices and to change the auction rules and formats to reduce the ability of bidders to collude.[28]

2.3 Revenue and Efficiency in the Incentive Auction

In February 2012, the U.S. Congress authorized the FCC to conduct an "incentive auction," a centralized two-sided market designed to transfer licenses from television broadcasters to providers of mobile wireless services.

The authorizing legislation for the incentive auction states that, in order for any transactions to occur, the sale of licenses to providers of wireless services must raise funds sufficient to cover: (i) the accepted bids of the television broadcasters, (ii) the FCC's out-of-pocket costs of

consumers and the national economy, the timely and economically advantageous development and sustained provision of UMTS services in the UK. Subject to this overall aim the Government's objectives are to (i) utilise the available UMTS spectrum with optimum efficiency; (ii) promote effective and sustainable competition for the provision of UMTS services; and (iii) subject to the above objectives, design an auction which is best judged to realise the full economic value to consumers, industry and the taxpayers of the spectrum." (Hamsard, 18 May 1998; reported in Binmore and Klemperer (2002)).

[27]Klemperer (2002a) discusses how collusion and a difficulty in attracting bidders in ascending auctions may have contributed to these mixed results. The paper also discusses how sealed bid auctions may be valuable for attracting bidders in asymmetric settings as they allow weak bidders to potentially win the auction and discusses how hybrid 'Anglo-Dutch' designs may strike a balance between efficiency and revenue. See also Klemperer (2002b) and Klemperer (2004).

[28]In an SMR format, opportunities for collusion can be reduced by using an auction format that limits information on bidder identities. The FCC took steps in this direction starting with its proposal for an "anonymous" auction format for Auction 66 (AWS-1) in 2006, and has since moved to an ascending-bid auction format that does not reveal the identities of bidders during the course of the auction (Marx (2006)). Denmark was the first to move away from the ascending auction SMR format and to adopt instead a sealed-bid auction format (Klemperer (2002a)). When bidders are asymmetric, sealed-bid formats can lead to situations where weaker bidders can win the auction against stronger ones. The potential for such outcomes can facilitate entry particularly in settings where the relative strength of bidders is known. In the auctioning of 4G licenses, most countries have now adopted a clock-proxy format. This auction format combines a combinatorial clock with a secondary sealed-bid stage. See Charles River & Associates Inc. and Market Design Inc. (1998), Porter et al. (2003), Banks, Olson, Porter, Rassenti, and Smith (2003), Ausubel, Cramton, and Milgrom (2006), Salant (2014), and Levin and Skrzypacz (2014) for discussion of combinatorial clocks and the clock-proxy design.

conducting the auction, and (iii) the expected reimbursement costs of broadcasters and certain other parties associated with the license reassignments occurring as part of the auction.[29] Although the legislation authorizing incentive auctions does not require that the FCC raise a minimum amount of revenue or that it maximize proceeds from the auction, statements made by both members of Congress and FCC Commissioners reveal that substantial revenue is expected from the auction.[30] This leaves the FCC in the position of needing to develop a centralized market that provides incentives for broadcasters to relinquish spectrum rights, "repacking" those who do not (reassigning them to different channels), and allocating the reformulated licenses to wireless service providers, all while generating revenue for the public.

Milgrom et al. (2012) offer a proposal for the key elements in the design of the incentive auction. The proposed two-sided mechanism has the following basic structure (see Milgrom et al., 2012, for greater detail). The auctioneer sets a requirement for net revenue. For each geographic area, the auctioneer sets a target quantity and runs a descending clock auction with the sellers, starting from a reserve price, in order to identify bidder-specific prices at which the target quantity is supplied and to identify the set of sellers that would supply units at those prices. Then for each area, the auctioneer runs an ascending clock auction with the buyers, starting from a reserve price, in order to identify prices at which the target quantity is demanded and to identify which buyers would be willing to demand units at those prices.

[29] Public Law 112-96, Section 6403(c)(2)(B), http://www.gpo.gov/fdsys/pkg/PLAW-112publ96/pdf/PLAW-112publ96.pdf.

[30] In a Congressional Hearing on "Keeping the New Broadband Spectrum Law on Track" (U.S. House Energy and Commerce Committee, 12 Dec. 2012), FCC Commissioner Robert McDowell stated, "The overarching goals of the law are to auction all reclaimed spectrum to offer consumers more opportunities to harness wireless broadband, while raising badly needed funds for the U.S. Treasury and attempt to fund a nationwide, interoperable, mobile broadband public safety network." In the same hearing, FCC Commissioner Ajit Pai argued that if the incentive auction did not yield any net revenues, "That would mean no money for the First Responder Network Authority (FirstNet) to build out a nationwide, interoperable public safety broadband network; no money for state and local first responders; no money for public safety research; no money for deficit reduction; and no money for next-generation 911 implementation. Most of the problem stems from the structure of the proposed auction. The only closing condition set forth in the Notice of Proposed Rulemaking is that the revenues from the forward auction must cover the costs of the reverse auction." In the question-and-answer portion of the hearing, the FCC Commissioners were asked, "Should the commission ensure that the auction raises $7b [for a nationwide interoperable public safety network]?" The responses were: "Pai: Yes, we should focus on maximizing revenue. Rosenworcel: Yes, absolutely. Clyburn: Absolutely. McDowell: Yes. Genachowski: Yes." According to the *New York Times* ("Republicans Tell F.C.C. Not to Give Away Airwaves," Edward Wyatt, 12 Dec. 2012), "Representative Greg Walden, an Oregon Republican who is chairman of the subcommittee on communications and technology, said that the law that gave the F.C.C. the ability to conduct 'incentive auctions' of newly available spectrum required 'maximizing the proceeds from the auction."' According to *Politico Pro* ("Terry: FCC's spectrum auction all about raising $24 billion," Tony Romm, 9 Jan. 2013), "Rep. Lee Terry emphasized the agency is under a strict mandate to raise some big bucks. 'Let's not fool ourselves, the major underlying maybe unstated reason for this auction is the money,' said the Nebraska Republican, speaking at the 2013 International CES. 'It was estimated we could raise $24 billion. That's not specifically laid out, but I can guarantee you that was part of the discussion. So we want the FCC to design the rules to get us at least $24 billion."'

The auctioneer can then calculate the net revenue associated with the supply-side and demand-side prices derived from the descending and ascending auctions. If that net revenue is above the required level, the auction ends, but if it is below the required level, then the auctioneer reopens the ascending clock auction, continuing to increase prices until either the net revenue hurdle is met or demand falls below the target quantity. If the hurdle is not met, then the auctioneer reduces the target quantity and reopens the descending and ascending auctions. For a given revenue requirement, it is possible that there is no quantity target for which the revenue requirement is satisfied.

The proposal recognizes that there are significant complexities associated with the purchase of spectrum licenses from broadcasters because spectrum released by broadcasters must be reformulated into a bandplan suitable for the provision of mobile wireless services and because broadcasters choosing not to sell their licenses must be reassigned to suitable spectrum. This process requires attention to a large number of interference constraints restricting which pairs of broadcast stations can be assigned to the same or adjacent channels. Theoretical foundations for the design proposed by Milgrom et al. (2012) are provided in Milgrom and Segal (2014), which discusses the computational and incentive properties of "deferred-acceptance" auctions. These are auctions that use an iterative process of scoring and rejecting bids, while checking that a set of constraints can continue to be satisfied. At each iteration, the least attractive bids are rejected, with all bids remaining at the end accepted. Milgrom and Segal (2014) show for a one-sided private-values environment in which bidders have single-unit supply that a deferred-acceptance auction satisfies individual and weak group strategy-proofness if it is a "threshold auction," i.e., the allocation rule is monotonic in the sense that if a seller's bid wins, then any lower bid would also win, and the payment to a winning seller is the maximum bid by that bidder that would be accepted, given the bids of the other bidders.[31] In addition, Milgrom and Segal (2014) show that the proposed descending clock-auction format for the buy side of the incentive auction can be used to implement a deferred-acceptance threshold auction, thereby addressing the computational complexity associated with the repurchase of spectrum while still providing incentives for truthful bidding.

The Milgrom et al. (2012) auction design recognizes that in a two-sided auction one may have to sacrifice efficiency in order to guarantee non-negative revenue by starting with a revenue target and quantity target and then iteratively reducing the quantity target (sacrificing

[31] The approach of Milgrom and Segal (2014) is also applicable to the two-sided mechanism of McAfee (1992), which we discuss below.

quantity) until the revenue target can be met. The proposed mechanism prioritizes efficiency over revenue by ending the auction at the quantity closest to the target quantity for which the revenue requirement is met. In addition, the details of the proposed design, which we have not delved into above, recognize the value in accounting for the potential steepness of the tradeoff between revenue and efficiency in a two-sided auction.[32]

Just as the call to design primary markets for spectrum licenses mobilized economists and spurred thinking, research, and debate on auctions, there is the potential for a similar effect in the wake of the call for the design of an incentive auction on research and debates pertaining to the design of two-sided market mechanisms.[33] The literature explicitly addressing incentive auctions of the type envisioned by the FCC is growing,[34] but many of the foundational results required to advance this literature remain to be developed.

3 Theoretical Background

3.1 Impossibility of Ex post Efficient Trade

A well-known and influential finding in mechanism design is the impossibility result of Myerson and Satterthwaite (1983), which states that under fairly general conditions, any efficient mechanism that satisfies the buyer's and seller's incentive compatibility and individual rationality constraints will generate a deficit for the mechanism designer. In order for the auctioneer to discover whether it is efficient for trade to occur, buyers must be induced to reveal the maximum price at which they are willing to buy, while sellers must be induced to reveal the minimum price at which they are willing to sell. Clearly, buyers and sellers will reveal this information only if telling the truth makes them better off than lying. As we discuss in greater detail in Section 4, generating the correct incentives for both buyers and sellers requires that

[32]As explained by Milgrom et al. (2012, p.19): "Additionally, the Closing Conditions could incorporate a trade-off between the amount of spectrum cleared and Net Revenue to ensure that the Commission does not give up too much spectrum clearing to attain slightly higher Net Revenue. If the increase in Net Revenue relative to the reduction in spectrum is too low, the auction would end at the previous (greater) Clearing Target. If the increase in Net Revenue relative to the reduction in spectrum is sufficiently large, the auction would continue with new, reduced Clearing Targets, until the clearing conditions are met."

[33]The FCC has hired teams of economists to advise the process: "To help design and implement incentive auctions the FCC retained leading experts in auction theory and implementation from Auctionomics and Power Auctions. The auction design team is composed of Professors Paul Milgrom, Jonathan Levin, and Ilya Segal of Stanford University, and Professor Lawrence Ausubel of the University of Maryland." (Kwerel, LaFontaine, and Schwartz, 2012, n.1)

[34]Existing papers include Bazelon, Jackson, and McHenry (2011), which estmates revenues from the sale of spectrum and the compensation required for broadcasters to sell their current broadcasting licenses, focusing on the issue of repacking broadcast licenses and concluding that a profitable auction is likely to be possible; and Mayo and Wallsten (2011), which discusses efficiency benefits of having both incentive auctions and well-functioning resale markets to ensure efficient spectrum use.

the purchase prices paid by buyers be below the sale prices paid to sellers, resulting in a deficit that must be paid by the mechanism designer in order to facilitate efficient trade.

As an illustration, consider the Myerson-Satterthwaite problem with a buyer and a seller whose types v and c are independently drawn from the continuous distributions $F(v)$ and $G(c)$, respectively. The densities of F and G, denoted $f(v)$ and $g(c)$ respectively, are positive everywhere on the identical support $[0, 1]$. Ex post efficiency means that the good changes hands if and only if $v \geq c$. Therefore, expected welfare W^* is

$$W^* := \int_0^1 \int_0^v (v - c) g(c) f(v) dc dv.$$

By the well-known revenue equivalence theorem (see e.g. Myerson (1981) and Krishna (2002)), the expected revenue from any mechanism that induces ex post efficient trade in equilibrium and respects individual rationality is no more than the expected revenue from a second-price double auction (a special case of the Vickrey or Vickrey-Clarke-Groves (VCG) mechanism).[35]

In the second-price double auction, the seller announces the minimum price at which he is willing to sell $\hat{c}$, and the buyer announces the maximum price at which she is willing to trade $\hat{v}$. If $\hat{v} > \hat{c}$, then the two parties trade and the buyer pays $\hat{c}$ while the seller receives $\hat{v}$. Because individuals' payments are independent of their values, the usual second-price logic implies that it is a dominant strategy for each individual to report truthfully. Thus, $\hat{v} = v$ and $\hat{c} = c$ and the expected revenue under this mechanism, denoted R^{VCG}, is

$$R^{VCG} := \int_0^1 \int_0^v (c - v) g(c) f(v) dc dv = -W^*.$$

In this example with identical supports, a subsidy equal to the expected welfare gain is necessary to induce ex post efficient trade. In the words of Vickrey (1961, p.8), the demands this would impose on the fiscal resources of the state, assuming that government is the market maker, would indeed seem "inordinately expensive."[36]

[35] The VCG mechanism is named after Vickrey (1961), Clarke (1971), and Groves (1973) and introduced formally in Section 4.2. To the best of our knowledge, Makowski and Ostroy (1987, p.246) coined the term "Vickrey-Clark-Groves (VCG)" mechanism, noting its remarkable genesis as a market mechanism which arose from independent proposals "to cope with allocational problems within a firm (Groves [1973]) or with monopoly problems (Vickrey [1961]) or public goods (Clarke [1971], Groves and Loeb [1975]) where the market does not operate or does not operate well." Makowski and Ostroy (1987, p.260) conclude with what they call a historical paradox: "Far from showing how a purposefully designed mechanism can do what competitive markets cannot, mechanism theory, by posing and solving the dominant strategy or revelation problem, has rediscovered and given a new articulation to the underlying logic of competitive market equilibrium! That is, it has rediscovered and rearticulated the importance of rewarding people with their marginal products."

[36] Our focus in this paper is on private goods. Using similar techniques, Rob (1989) and Mailath and Postlewaite (1990) have established impossibility results for public goods with particularly dire implication when economies become large. A more positive message emerges from the papers by Neeman (1999) and Hellwig (2003), who establish conditions on initial property rights and the nature of the public good, respectively, under which ex post efficiency is possible.

Broadly based on such impossibility results, the economics literature, and the practice of market design, have focused on the design of one-sided markets and auctions (see, for example, Milgrom (2004, Chapter 1)). The reasoning is simple and compelling: in a one-sided market, a mechanism can allocate objects to the agents with the highest values while at the same time generating a surplus. After such an efficient mechanism has been run, there is no scope for secondary market transactions.

However, getting the allocation right in the first place may not always be an option. For example, unforeseen and unforeseeable technological change such as the advent of mobile telephony and the internet may render an initially efficient allocation of resources socially undesirable. If the original property rights are permanent, then without depriving some owners of their rights, it is impossible to allocate these assets more efficiently without relying on some kind of secondary market. This raises the question as to what guidance the economics literature can provide for the design of such markets.

3.2 Constrained Efficient Mechanisms

While the impossibility results of Myerson and Satterthwaite (1983) are well known, the paper also highlights the possibility of constrained efficient mechanisms that do not run a deficit and improve efficiency relative to having no centralized secondary market. The existence of such *possibility* results is likely to be important for practical design.

As an illustration, re-consider the Myerson and Satterthwaite problem discussed above and assume in addition that F and G are both uniform on $[0, 1]$. Assume that the buyer and seller participate in the double auction of Chatterjee and Samuelson (1983), whose rules are that the buyer and seller submit bids simultaneously and that the good changes hands if and only if the buyer's bid exceeds the seller's. The buyer pays an amount to the seller equal to the average of her bid and the seller's bid if trade occurs and nothing otherwise. Because the payment is simply a transfer from the buyer to the seller, the mechanism is budget-balanced ex post. It is also ex post individually rational because both the buyer and the seller can guarantee nonnegative payoffs by bidding below and above their values, respectively. In the linear equilibrium of this auction, the good changes hands if and only if $v \geq c + 1/4$, which results in a welfare of $W^{SB} = 9/64$.[37] Observe that for F and G uniform, $W^* = 1/6$. Thus,

[37]As shown by Myerson and Satterthwaite (1983), for the uniform-uniform case the double auction is a second-best mechanism in the sense of maximizing expected welfare subject to budget balance and incentive and individual rationality constraints.

most of the first-best welfare can be captured by a revenue-neutral second-best mechanism. Stated differently, the cost of avoiding a deficit is only 16% of the first-best welfare.[38]

Our discussion thus far has focused on environments in which goods are indivisible and ownership shares are either zero or one. However, there are also environments in which partial ownership is possible. For the case of partial ownership shares, another important possibility result has been obtained by Cramton, Gibbons, and Klemperer (1987). They show that the joint ownership of an asset can be resolved ex post efficiently with an incentive compatible, interim individually rational mechanism if the initial ownership shares are sufficiently close to equal. Intuitively, dispersed initial ownership shares reduce the incentives for misrepresentation because agents may end up as either buyers or sellers.

The work by Cramton, Gibbons, and Klemperer (1987) and the subsequent literature on partnership dissolution, such as Fieseler, Kittsteiner, and Moldovanu (2003), Schweizer (2006), Segal and Whinston (2011), and Figueroa and Skreta (2012), brings to light a tradeoff that is potentially important in the context of allocating long-run property rights to an asset. To be concrete, suppose there is an object to be allocated to a set of agents each of whom may own partial shares. In a static environment, efficiency dictates that it be allocated to the agent with the highest value. However, in a dynamic environment, changes over time such as the emergence of a new technology can alter the efficient allocation, reducing long-run efficiency if efficiency re-allocation is not feasible. In fact, from the results by Myerson and Satterthwaite (1983) and Cramton, Gibbons, and Klemperer (1987), we know that an ex post efficient re-allocation is feasible only if the initial allocation was not efficient, that is, only if initially full ownership was not given to the agent who initially had the highest value.[39] Of course, in order to address the question of what initial allocation would initially be optimal, one would need to know the set of the constrained efficient mechanisms with ownership shares that are not simply zero or one. To the best of our knowledge, this question has not been addressed in this literature.

4 Revenue and Efficiency with Two-Sided Private Information

In this section, we lay out the underpinnings of market design with privately informed buyers and sellers. We introduce in Section 4.1 the basic setup with homogenous goods and buyers

[38]The 16% is calculated as $16\% = (1/6 - 9/64)/(1/6)$.

[39]Evidence of exactly this dynamic misallocation is provided by Bleakley and Ferrie (2014).

that have unit demand and sellers that have unit capacity. In Section 4.2, we show that the impossibility result of Myerson-Satterthwaite generalizes to a setting with an arbitrary number of buyers and sellers. In contrast to one-sided problems where efficient mechanisms also generate revenue, efficiency and positive revenue are not possible in two-sided markets. We discuss the revenue-efficiency tradeoff for two-sided markets in Section 4.3, and then in Section 4.4, we contrast the revenue-efficiency tradeoff for two-sided markets with that for one-sided markets.

4.1 Setup

We consider a setup with B buyers $b \in \mathbb{B}$ and S sellers $s \in \mathbb{S}$, where $\mathbb{B}$ and $\mathbb{S}$ denote, respectively, the sets of buyers and sellers. As is standard in the literature on Bayesian mechanism design, we assume buyers have unit demand and sellers have unit supply of a homogeneous good.

We assume quasilinear payoff functions. That is, if buyer b receives a unit and makes the transfer payment T_b, her payoff is $v_b - T_b$, where v_b is her valuation for a unit. Similarly, if seller s produces a unit and receives the transfer T_s his payoff is $T_s - c_s$, where c_s denoting his cost. We assume that v_b and c_s are private information of b and s for all $b \in \mathbb{B}$ and for all $s \in \mathbb{S}$. The valuation of receiving and the cost of producing nothing is normalized to 0. Sometimes we will refer to v_b and c_s as buyer b's and seller s's type, respectively.

Let $\underline{v}$ and $\overline{v}$ be, respectively, the lowest and highest possible valuation of every buyer and let $\underline{c}$ and $\overline{c}$ be, respectively, the lowest and highest possible cost of every seller for producing a unit. We assume that

$$\underline{v} \leq \underline{c} \quad \text{and} \quad \overline{v} \leq \overline{c} \tag{1}$$

and that these bounds are common knowledge; in particular, they are known to the mechanism designer. Condition (1) implies that, depending on the buyers' and the sellers' types, no trade at all can be optimal. This condition guarantees that the least efficient buyer and seller types never trade under an efficient allocation rule.[40]

To make the problem interesting, we also assume

$$\underline{c} < \overline{v}, \tag{2}$$

[40] Absent this condition, it is well known that ex post efficient trade without a deficit may be possible. Makowski and Mezzetti (1993) observe the possibility of ex post efficiency without a deficit in a model with multiple buyers and one seller. Makowski and Mezzetti (1994) provide a general characterization of ex post efficient Bayesian mechanisms. Williams (1999) and Schweizer (2006) derive conditions for the possibility of ex post efficiency without a deficit for a model with multiple buyers and sellers in relation to the ordering of the bounds of the supports and the numbers of buyers and sellers.

which implies that for some types of buyers and sellers, some trade is efficient. We also assume that buyers' types and sellers' types are drawn independently from continuous distributions whose densities are positive everywhere on their respective supports $[\underline{v}, \overline{v}]$ and $[\underline{c}, \overline{c}]$.

4.2 Impossibility of Ex Post Efficient Trade without Running a Deficit

The well-known revelation principle (Myerson (1981)) implies that without loss of generality we can restrict our attention to direct mechanisms, that is, mechanisms according to which every agent is simply asked to report his type to a mechanism that satisfies agents' incentive compatibility and individual rationality constraint, thereby ensuring that agents have no incentive to misreport their types. We use this result and consider mechanisms that are defined as an allocation rule $\mathbf{Q}(\mathbf{v}, \mathbf{c})$ and a transfer rule $\mathbf{T}(\mathbf{v}, \mathbf{c})$ that both depend on the vector of reported types $(\mathbf{v}, \mathbf{c})$. An allocation rule $\mathbf{Q}(\mathbf{v}, \mathbf{c})$ specifies for every buyer b and seller s the probability $Q_b(\mathbf{v}, \mathbf{c})$ that buyer b receives a unit of the good and the probability $Q_s(\mathbf{v}, \mathbf{c})$ that seller s produces the good in equilibrium when the types are $(\mathbf{v}, \mathbf{c})$. The payment rule $\mathbf{T}(\mathbf{v}, \mathbf{c})$ specifies an expected transfer $T_b(\mathbf{v}, \mathbf{c})$ paid by each buyer b and an expected transfer $T_s(\mathbf{v}, \mathbf{c})$ received by each seller. We restrict attention to feasible allocations where the sellers production is equal to or exceeds buyers consumption for all $(\mathbf{v}, \mathbf{c})$ and for all resolutions of uncertainty.

The expected welfare of a mechanism $\langle \mathbf{Q}, \mathbf{T} \rangle$ is given by

$$W_{\mathbf{Q}} := E\left[\sum_b v_b Q_b(\mathbf{v}, \mathbf{c}) - \sum_s c_s Q_s(\mathbf{v}, \mathbf{c})\right] \tag{3}$$

while the expected revenue of this mechanism is

$$R_{\mathbf{T}} := E\left[\sum_b T_b(\mathbf{v}, \mathbf{c}) - \sum_s T_s(\mathbf{v}, \mathbf{c})\right]. \tag{4}$$

We define $\mathbf{Q}^*(\mathbf{v}, \mathbf{c})$ as the maximizer of $W_{\mathbf{Q}}$. We further define $W^*(\mathbf{v}, \mathbf{c})$ as the maximized value of social welfare, $W^*_{-b,.}(\mathbf{v}, \mathbf{c})$ the maximized value of social welfare with buyer b removed, $W^*_{.,-s}(\mathbf{v}, \mathbf{c})$ the maximized social welfare when seller s is removed, and $W^*_{-b,-s}(\mathbf{v}, \mathbf{c})$ the maximized social welfare when the pair b and s are removed.

A second well-known result, often called the revenue equivalence theorem, states that under general conditions, which are satisfied for our environment, the expected payoff of any agent of any type will differ across two incentive compatible mechanisms with the same allocation rule $\mathbf{Q}(\mathbf{v}, \mathbf{c})$ by at most a constant.[41] Consider the VCG mechanism, so named after the

[41] A first instance of revenue equivalence was noticed by Vickrey (1961, 1962). Myerson (1981) and Riley and Samuelson (1981) provide general formulations and formalization. The revenue equivalence theorem we are invoking here is due to Krishna and Maenner (2001); see also Krishna (2002).

independent contributions by Vickrey (1961), Clarke (1971), and Groves (1973). The VCG mechanism is a direct mechanism that uses an efficient allocation rule $\mathbf{Q}^*(\mathbf{v}, \mathbf{c})$. The transfer payment T_b from buyer b to the mechanism is equal to the net expected welfare lost by the other individuals due to her participation

$$T_b(\mathbf{v}, \mathbf{c}) := W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) - \left[W^*(\mathbf{v}, \mathbf{c}) - v_b Q^*_b(\mathbf{v}, \mathbf{c})\right]. \tag{5}$$

Likewise, the transfer to seller s by the mechanism is equal to the net expected welfare generated by his participation

$$T_s(\mathbf{v}, \mathbf{c}) := \left[W^*(\mathbf{v}, \mathbf{c}) + c_s Q^*_s(\mathbf{v}, \mathbf{c})\right] - W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}). \tag{6}$$

The VCG mechanism is incentive compatible because every agent has a dominant strategy to report his type truthfully. It is also individually rational because $v_b - T_b(\mathbf{v}, \mathbf{c}) \geq 0$ and $T_s(\mathbf{v}, \mathbf{c}) - c_s \geq 0$ for any $b \in \mathbb{B}$ and any $s \in \mathbb{S}$ and any $(\mathbf{v}, \mathbf{c})$.

The revenue equivalence theorem implies that all mechanisms with the same allocation rule have the same payment rule up to a constant. This constant corresponds to the payment given to buyers and sellers who do not trade in equilibrium. Condition (1) implies that the least efficient type of a seller (that is, a seller whose cost of production is $\overline{c}$) and the least efficient type of a buyer (that is, a buyer who values a unit at $\underline{v}$) never trade under an efficient allocation rule. Given that the VCG mechanism is based on such an allocation rule, and $W_{-b,\cdot}(\mathbf{v}, \mathbf{c}) = W(\mathbf{v}, \mathbf{c})$ and $W_{\cdot,-s}(\mathbf{v}, \mathbf{c}) = W(\mathbf{v}, \mathbf{c})$ for any b and s that do not trade, it follows that the payments to and from such agents are 0, and so are their payoffs.

A consequence of the revenue equivalence theorem is, therefore, that any other incentive compatible, individually rational and efficient mechanism generates weakly less revenue than $R(\mathbf{v}, \mathbf{c}) = \sum_b T_b(\mathbf{v}, \mathbf{c}) - \sum_s T_s(\mathbf{v}, \mathbf{c})$ when the types are $(\mathbf{v}, \mathbf{c})$. Thus, to prove that, under conditions that are to be specified, it is impossible to allocate objects efficiently while respecting agents' individual rationality and incentive compatibility constraints without running a deficit, it is sufficient to show that the VCG mechanism runs a deficit.[42] This is what we do now.

Without loss of generality, we consider a VCG mechanism that orders the buyers values from highest to lowest and the sellers from lowest cost to highest cost and matches buyers and sellers into pairs until all pairs where the buyer's value exceeds the seller's cost are exhausted.

[42] The proof of the Myerson-Satterthwaite (1983) impossibility result using revenue equivalence was developed by Williams (1999) and independently by Krishna and Perry (2000), with awareness of the argument evident in Makowski and Mezzetti (1994). For an alternative approach and generalization, see Makowski and Ostroy (1989) and the extension by Segal and Whinston (2012).

Much like the construction of supply and demand graphs, this approach will always generate an efficient allocation because the buyers with the highest values and the sellers with the lowest costs will trade.

Because the allocation is specified by a set of bilateral trading pairs, a useful way of expressing the final allocation of goods is by describing the final allocation by a network of trading links in the bipartite graph of buyers and sellers. Let L^* denote the network of trading links that induces the allocation $\mathbf{Q}^*$ in the way described above whose typical element is $l^*_{bs} \in \{0, 1\}$ with $l^*_{bs} = 1$ meaning seller s produces a unit of the good for buyer b and $l^*_{bs} = 0$ meaning that seller s produces nothing for buyer b.

If a pair of individuals who are trading in equilibrium are removed from the market, the allocations to all the other individuals are unchanged. This implies that total welfare can be calculated by adding the utility generated from each individual pair:

$$\sum_{l^*_{bs} \in L^*} (W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c})) = W^*(\mathbf{v}, \mathbf{c}). \tag{7}$$

Having shown that welfare can be calculated by adding up the welfare of each trading pair, we now need to show that the transfers received by the seller in each pair will exceed the transfer paid by each buyer. We do this by arguing that buyers and sellers are complements. For any model with one-to-one matching, Shapley (1962) showed that

$$W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c}) + W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c}) \leq W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c}) \tag{8}$$

for all $b \in \mathbb{B}, s \in \mathbb{S}$. This condition has a straightforward interpretation. The expressions $W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c})$ and $W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c})$ capture, respectively, the individual marginal contribution to welfare of buyer b and seller s to an economy that consists of all buyers other than b and all sellers other than s. The right side of (8) is the marginal contribution of adding the pair consisting of b and s to the economy without this pair. Condition (8) then simply states that the marginal contribution of the pair is not less than the sum of the individual marginal contributions.[43] As the next theorem shows, the fact that Shapley's complement condition (8) and condition (7) hold immediately implies that ex post efficient trade without running a deficit is impossible.[44]

[43]See Makowski and Ostroy (1987) and Makowski and Ostroy (2001) for an analysis of mechanisms in which each agent is always paid exactly his marginal contribution.

[44]Williams (1999, Theorem 4 and Table 1) shows that the result of Theorem 1 is sensitive to assumptions on the supports of the distributions from which buyers and sellers draw their values and costs. Recall that we assume that $\underline{v} \leq \underline{c}$ and $\overline{v} \leq \overline{c}$, guaranteeing overlapping support for the range of values and costs where trade generates surplus. This corresponds to row 1 in Table 1 of Williams (1999).

Theorem 1 *Assume that buyers' and sellers' types are drawn independently from distributions with positive densities everywhere on supports defined in equations* (1) *and* (2)*. In any environment where conditions* (7) *and* (8) *hold, it is impossible to allocate goods efficiently via an incentive compatible and individually rational mechanism without running a deficit in expectation. Moreover, given an efficient, individually rational, and incentive compatible mechanism, there is no realization* $(\mathbf{v}, \mathbf{c})$ *such that the mechanism produces a surplus for that realization.*

Proof of Theorem 1: As argued in the text, to prove the second part of the theorem, it suffices to show that the VCG mechanism never runs a surplus, i.e., $R_{\mathbf{T}}(\mathbf{v}, \mathbf{c}) \leq 0$ for all realizations of $(\mathbf{v}, \mathbf{c})$. We start by proving this result and then use it to prove the first part of the theorem.

By definition, $R_{\mathbf{T}}(\mathbf{v}, \mathbf{c}) = \sum_b T_b(\mathbf{v}, \mathbf{c}) - \sum_s T_s(\mathbf{v}, \mathbf{c})$. Using the definitions of $T_b(\mathbf{v}, \mathbf{c})$ and $T_s(\mathbf{v}, \mathbf{c})$ from equations (5) and (6) and noting that $\sum_b v_b Q_b^*(\mathbf{v}, \mathbf{c}) - \sum_s c_b Q_s^*(\mathbf{v}, \mathbf{c}) = W^*(\mathbf{v}, \mathbf{c})$, the condition $R_{\mathbf{T}}(\mathbf{v}, \mathbf{c}) \leq 0$ is equivalent to requiring that

$$\sum_b (W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c})) + \sum_s (W^*(\mathbf{v}, \mathbf{c}) - W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c})) \geq W^*(\mathbf{v}, \mathbf{c}). \tag{9}$$

Because $W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) = 0$ for a buyer who does not trade and $W^*(\mathbf{v}, \mathbf{c}) - W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}) = 0$ for a seller who does not trade, the left side of equation (9) can be written as $\sum_{l^*_{bs} \in L^*} [W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) + W^*(\mathbf{v}, \mathbf{c}) - W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c})]$. Further, condition (7) implies that the right side is equal to $\sum_{l^*_{bs} \in L^*} (W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c}))$. Thus, if we can show that for each trading pair

$$W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) + W^*(\mathbf{v}, \mathbf{c}) - W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}) \geq W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c}) \tag{10}$$

we are done.

To see that this condition is true under condition (8), we show that the opposite inequality leads to a contradiction for any b and s. Suppose

$$W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) + W^*(\mathbf{v}, \mathbf{c}) - W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}) < W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c}). \tag{11}$$

Inequality (8) is equivalent to $W^*_{\cdot,-s}(\mathbf{v}, \mathbf{c}) \leq W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) + W^*_{-b,-s}(\mathbf{v}, \mathbf{c})$. We thus conclude that the left side of (11) is not less than $W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) + W^*(\mathbf{v}, \mathbf{c}) - (W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,\cdot}(\mathbf{v}, \mathbf{c}) + W^*_{-b,-s}(\mathbf{v}, \mathbf{c})) = W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c})$, which is the expression on the right side of (11) and thus delivers the desired contradiction.

Because we have shown that the VCG mechanism never runs a surplus for any realization of $(\mathbf{v}, \mathbf{c})$, the mechanism will run a deficit in expectation if there exists at least one realization of $(\mathbf{v}, \mathbf{c})$ where the mechanism runs a deficit. Consider the case where one buyer has a value $\overline{v}$ and the other buyers have values $\underline{v}$ and one seller has value $\underline{c}$ and the other sellers have values $\overline{c}$. In this case, only the high valued buyer and low cost seller trade, the high valued buyer pays $\underline{c}$ and the low cost seller receives $\overline{v}$. By the assumption in equation (2), the mechanism generates a deficit in this state. ■

As can be seen from the construction of the proof above, the impossibility result holds for a broader set of environments than the simple homogenous environment discussed here. For example, the assignment model of Shapley and Shubik (1972), where sellers can produce a single good and buyers have heterogeneous values over the goods of the sellers, satisfies both conditions and thus never runs a surplus. The matching model of Shapley (1962), which assumes one-to-one matchings but imposes no specific restrictions on the surplus function of any matched pair, also satisfies both conditions. It is an open question under what assumptions on primitives these conditions generalize to many-to-many settings.[45,46]

4.3 Revenue versus Efficiency in Two-Sided Markets

An implication from the preceding analysis and discussion is that in designing mechanisms for markets with two-sided private information, the tradeoff between revenue and efficiency is likely to be more salient than when information pertains to one side only. Unless the designer of an auction is willing to run a deficit, efficient two-sided mechanisms are typically not possible. Further, since information rents must be paid to both sides of the market in a two-sided problem, any attempt at increasing efficiency will require transfers to both sides of the market. This suggests that the tradeoff between revenue and efficiency will be steeper in a two sided setting than in a one-sided setting.

In order to better understand the tradeoff of revenue and efficiency in two-sided setting, we characterize the frontier of constrained efficient mechanisms and show that a class of mechanisms with an α-allocation rule can achieve any point along this frontier. Using characteristics of these α-allocation rules, we then compare the tradeoff of revenue and efficiency in the one-sided and two-sided cases.

[45] For setups with a monopoly on one side and a many-to-one matching technology, Bikhchandani and Ostroy (2006) show that the agents on the side with multiple agents are substitutes to each other.

[46] In models with many-to-many matchings, it is useful to replace condition (7) with $\sum_{l^*_{bs} \in L^*} (W^*(\mathbf{v}, \mathbf{c}) - W^*_{-b,-s}(\mathbf{v}, \mathbf{c})) \geq W^*(\mathbf{v}, \mathbf{c})$ in Theorem 1. The resulting modified theorem can accommodate decreasing marginal values and increasing marginal costs. The proof of the modified theorem is nearly identical to the one shown here.

We begin by adapting the standard concept of virtual valuations used in auctions (see e.g. Riley, 2012, chapter 12) to the two-sided setup. For an $\alpha \in [0, 1]$, define

$$J^B(v, \alpha) := v - \alpha \frac{1 - F(v)}{f(v)} \quad \text{and} \quad J^S(c, \alpha) := c + \alpha \frac{G(c)}{g(c)}. \tag{12}$$

The function $J^B(v, \alpha)$ can be called the weighted virtual valuation of the buyer, while the function $J^S(c, \alpha)$ has the interpretation of a weighted virtual cost of the seller. Observe first that $J^B(v, 0) = v$ and $J^S(c, 0) = c$ correspond to the true types, while $J^B(v, 1) = v - (1 - F(v))/f(v)$ is the well-known concept of a buyer's virtual valuation and $J^S(c, 1) = c + G(c)/g(c)$ is the somewhat less familiar concept of a seller's virtual cost. As noted by Bulow and Roberts (1989), $J^B(v, 1)$ and $J^S(c, 1)$ can be interpreted, respectively, as a buyer's marginal revenue and a seller's marginal cost, treating the (change in the) probability of trade as the (marginal change in) quantity. Notice also that $J^B(v, \alpha)$ and $J^S(c, \alpha)$ are convex combinations of the true and the virtual types, with weight α attached to the virtual types. We restrict attention to the regular case by assuming that $J^B(v, 1)$ and $J^S(c, 1)$ are strictly monotone in their arguments.

We say that a mechanism with an allocation rule $\mathbf{Q}(\mathbf{v}, \mathbf{c})$ is *constrained efficient* if it maximizes the expectation of welfare $W_{\mathbf{Q}}(\mathbf{v}, \mathbf{c}) := \sum_b v_b Q_b(\mathbf{v}, \mathbf{c}) - \sum_s c_s Q_s(\mathbf{v}, \mathbf{c})$ subject to some feasible minimum expected revenue $\underline{R}$, and subject to agents' incentive and individual rationality constraints.

For the usual reasons, the revelation principle applies. This implies that without loss of generality we can focus the search for constrained efficient mechanisms on direct mechanisms that ask each player to report his or her type, providing incentives to do so via incentive compatibility constraints and with incentives to participate in the mechanism via the interim individual rationality constraints. Thus, the constrained maximization problem is to choose $\mathbf{Q}$ to maximize

$$\max_{\mathbf{Q}} E[W_{\mathbf{Q}}(\mathbf{v}, \mathbf{c})] \quad \text{s.t.} \quad E[R_{\mathbf{Q}}(\mathbf{v}, \mathbf{c})] \geq \underline{R} \tag{13}$$

and subject to incentive compatibility and individual rationality constraints.

As with one-sided auction problems, incentive compatibility and individual rationality put structure on the transfers that must be made from buyers and to sellers as a function of their types. The following lemma characterizes the maximal revenue that can be generated from any incentive compatible and individual rational mechanism with allocation rule $\mathbf{Q}$.

Lemma 1 *The maximum expected revenue of an incentive compatible, individually rational* *mechanism with allocation rule* $\mathbf{Q}$ *is given by:*

$$E[R_{\mathbf{Q}}(\mathbf{v},\mathbf{c})] := E\left[\sum_{b\in\mathbb{B}} J^B(v_b,1)Q_b(\mathbf{v},\mathbf{c}) - \sum_{s\in\mathbb{S}} J^S(c_s,1)Q_s(\mathbf{v},\mathbf{c})\right]. \tag{14}$$

Proof of Lemma 1: Let $q_b(v_b) := E[Q_b(\mathbf{v},\mathbf{c})]$ and $q_s(c_s) := E[Q_s(\mathbf{v},\mathbf{c})]$ be the interim (expected) probability of trade under a mechanism with allocation rule $\mathbf{Q}(\mathbf{v},\mathbf{c})$, with the expectations being taken with respect to the types of all agents other than buyer b's and seller s's, respectively. Similarly, let $t_b(v_b) := E[T_b(\mathbf{v},\mathbf{c})]$ and $t_s(c_s) := E[T_s(\mathbf{v},\mathbf{c})]$. The expected utility of a buyer of type v_b is given by $U_b(v_b) := v_b q_b(v_b) - t_b(v_b)$, while the expected utility of a seller is given by $U_s(c_s) := t_s(c_s) - c_s q_s(c_s)$.

By a standard envelope theorem argument (see e.g. Krishna, 2002), incentive compatibility implies

$$U_b(v_b) = U_b(\underline{v}) + \int_{\underline{v}}^{v_b} q_b(y)dy \tag{15}$$

and

$$U_s(c_s) = U_s(\overline{c}) + \int_{c_s}^{\overline{c}} q_s(y)dy. \tag{16}$$

Thus, every agent's expected payoff is pinned down, up to a constant, which is $U_b(\underline{v})$ if she is a buyer and $U_s(\overline{c})$ if he is a seller, by the allocation rule. Because the integral terms are non-negative and equal to zero for $v_b = \underline{v}$ and $c_s = \overline{c}$, interim individual rationality is satisfied if and only if $U_b(\underline{v}) \geq 0$ and $U_s(\overline{c}) \geq 0$.

Using the definitions $U_b(v_b)$ and $U_s(c_s)$ and noting that the least efficient types, $\underline{v}$ and $\overline{c}$, never trade, the expected payment from a buyer of type v_b and the expected payment to a seller of type c_s are

$$t_b(v_b) = t_b(\underline{v}) + v_b q_b(v_b) - \int_{\underline{v}}^{v_b} q_b(y)dy \tag{17}$$

and

$$t_s(c_s) = t_s(\overline{c}) + c_s q_s(c_s) - \int_{c_s}^{\overline{c}} q_s(y)dy. \tag{18}$$

From an *ex ante* perspective, the expected transfer from a buyer of unknown type (expressed as the random variable V) is

$$E[t_b(V)] = \int_{\underline{v}}^{\overline{v}} t_b(x)f(x)dx. \tag{19}$$

Substituting for $t_b(x)$ in the equation above using (17), integrating by parts twice and using the definition of $J^B(v,\alpha)$ from (12) results in an expected transfer from each buyer of

$$E[t_b(V)] = t_b(\underline{v}) + \int_{\underline{v}}^{\overline{v}} J^B(y,1)q_b(y)f(y)dy. \tag{20}$$

By a similar substitution, the expected transfer to each seller is

$$E[t_s(C)] = t_s(\overline{c}) + \int_{\underline{c}}^{\overline{c}} J^S(y,1)q_s(y)g(y)dy. \tag{21}$$

Using the definition of $q_b(v_b)$ and $q_s(c_s)$ and noting that $R = \sum_{b\in\mathbb{B}} E[t_b(V)] - \sum_{s\in\$} E[t_s(C)]$, it follows that the expected revenue R of an incentive compatible mechanism $\langle \mathbf{Q}, \mathbf{T}\rangle$ is

$$R = E\left[\sum_{b\in\mathbb{B}} J^B(v_b,1)Q_b(\mathbf{v},\mathbf{c}) - \sum_{s\in\$} J^S(c_s,1)Q_s(\mathbf{v},\mathbf{c})\right] + \sum_{b\in\mathbb{B}} t_b(\underline{v}) - \sum_{s\in\$} t_s(\underline{c}). \tag{22}$$

Interim individual rationality requires that $U_b(\underline{v}) = -t_b(\underline{v}) \geq 0$ and $U_s(\overline{c}) = t_s(\overline{c}) \geq 0$. Thus, the maximal revenue occurs when these transfer payments are zero. ■

Using the constraints that incentive compatibility places on the optimization problem, an alternative formulation of the constrained optimization problem is

$$\max_{\mathbf{Q}} E[W_{\mathbf{Q}}(\mathbf{v},\mathbf{c})] \quad \text{s.t.} \quad E\left[\sum_{b\in\mathbb{B}} J^B(v_b,1)Q_b(\mathbf{v},\mathbf{c}) - \sum_{s\in\$} J^S(c_s,1)Q_s(\mathbf{v},\mathbf{c})\right] \geq \underline{R}. \tag{23}$$

Letting $\lambda \in [0,\infty)$ be the Lagrangian on the revenue constraint, the constrained optimization problem can be written as

$$\max_{\mathbf{Q}} E\left[\sum_{b\in\mathbb{B}} (v_b + \lambda(\underline{R})J^B(v_b,1))Q_b(\mathbf{v},\mathbf{c}) - \sum_{s\in\$} (c_s + \lambda(\underline{R})J^S(c_s,1))Q_s(\mathbf{v},\mathbf{c})\right] - \lambda(\underline{R})\underline{R}. \tag{24}$$

Because $\lambda(\underline{R}) \geq 0$, we can divide through equation (24) by $\frac{1}{1+\lambda(\underline{R})}$ without changing the optimal mechanism. Letting $\alpha(\underline{R}) = \frac{\lambda(\underline{R})}{1+\lambda(\underline{R})}$ and noting that $(1-\alpha(\underline{R})) = \frac{1}{1+\lambda(\underline{R})}$, an equivalent maximization problem is

$$\max_{\mathbf{Q}} E\left[\sum_{b\in\mathbb{B}} J^B(v_b,\alpha(\underline{R}))Q_b(\mathbf{v},\mathbf{c}) - \sum_{s\in\$} J^S(c_s,\alpha(\underline{R}))Q_s(\mathbf{v},\mathbf{c})\right] - \alpha(\underline{R})\underline{R}. \tag{25}$$

For a fixed α, equation (25) can be optimized for any $(\mathbf{v},\mathbf{c})$ by ordering the buyers from highest weighted virtual valuation to lowest virtual valuation and sellers from lowest virtual cost to highest virtual cost and then matching corresponding buyers with sellers as long as the buyer's virtual valuation exceeds the seller's virtual cost (see Loertscher and Niedermayer (2013)).[47] Much like the construction of standard supply and demand graphs, this approach will always maximize equation (25) because the buyers with the highest weighted virtual valuations will trade, the sellers with the lowest weighted virtual costs will trade, and all pairs that

[47] Given continuous distributions, the buyer and seller orderings will be unique with probability 1. In case of non-uniqueness, one can arbitrarily order buyers with the same valuation and sellers with the same cost.

generate a negative weighted virtual surplus will be excluded. The following lemma characterizes the allocation rule of constrained-efficient, incentive compatible and individually rational mechanisms using this logic.

Lemma 2 *Let* $\mathbb{B}^*_\alpha(\mathbf{v},\mathbf{c})$ *and* $\mathbb{S}^*_\alpha(\mathbf{v},\mathbf{c})$ *be the sets of buyers and sellers that would trade under an efficient rule if the true types were* $J^B(v_b,\alpha)$ *for all* $b\in\mathbb{B}$ *and* $J^S(c_s,\alpha)$ *for all* $s\in\mathbb{S}$ *when the types are* $(\mathbf{v},\mathbf{c})$*. Any constrained efficient mechanism corresponds to a mechanism in which a buyer* b *trades if an only if* $b\in\mathbb{B}^*_\alpha(\mathbf{v},\mathbf{c})$ *and a seller* s *trades if an only if* $s\in\mathbb{S}^*_\alpha(\mathbf{v},\mathbf{c})$*, for some value of* $\alpha\in[0,1]$*.*

Proof of Lemma 2: By construction, every solution to the constrained optimization problem in (13) is a solution to (24) for some $\lambda\in[0,\infty)$. Because α is a monotonic transformation of λ, every solution to the constrained optimization problem in (13) is also a solution to (25) for $\alpha(\underline{R})\in[0,1)$.

Because the mechanism described in Lemma 2 maximizes the expression in (25) for each $(\mathbf{v},\mathbf{c})$, it also maximizes its expected value for any α. We are thus left to verify that this allocation rule implies monotonicity of $q_b(v_b)$ and $q_s(c_s)$. Given the imposed monotonicity of the functions $J^B(v_b,1)$ and $J^S(c_s,1)$, $q_b(v_b)$ is increasing and $q_s(c_s)$ decreasing because the probability of belonging to the sets of agents who trade under efficiency when the true types are $J^B(.,\alpha)$ and $J^S(.,\alpha)$ is increasing in v_b and decreasing in c_s. ■

Following Gresik and Satterthwaite (1989), we refer to the set of allocation rules described in Lemma 2 as α-allocation rules because the α parameter has an intuitive interpretation as the weight placed on revenue for a social planner maximizing a weighted sum of revenue and efficiency and the parameter α can fully describe variation in the optimal rules along the frontier.

Under a constrained-efficient mechanism with a given a α-allocation rule, expected welfare $W(\alpha)$ and revenue $R(\alpha)$ are:

$$W(\alpha) := E\left[\sum_{b\in\mathbb{B}^*_\alpha(\mathbf{v},\mathbf{c})} v_b - \sum_{s\in\mathbb{S}^*_\alpha(\mathbf{v},\mathbf{c})} c_s\right] \tag{26}$$

and

$$R(\alpha) := E\left[\sum_{b\in\mathbb{B}^*_\alpha(\mathbf{v},\mathbf{c})} J^B(v_b,1) - \sum_{s\in\mathbb{S}^*_\alpha(\mathbf{v},\mathbf{c})} J^S(c_s,1)\right], \tag{27}$$

where expectations are taken with respect to the densities $f(\mathbf{v})$ and $g(\mathbf{c})$. For $\alpha \in [0,1]$, $W(\alpha)$ is strictly decreasing in α while $R(\alpha)$ is strictly increasing in α. This is the trade-off between efficiency and revenue. Observe also that $W(0)$ is the maximal level of welfare, while $R(1)$ is the maximal level of revenue that can be obtained in the setup with two-sided private information.

As an illustration, consider the bilateral trade problem of Myerson and Satterthwaite (1983) with F and G uniform on $[0,1]$. Then $J^B(v,\alpha) = (1+\alpha)v - \alpha$ and $J^S(c,\alpha) = (1+\alpha)c$, and so $W(\alpha) = \frac{1+3\alpha}{6(1+\alpha)^3}$ and $R(\alpha) = \frac{-1+3\alpha}{6(1+\alpha)^3}$. It can be seen that under the efficient mechanism, welfare $W(0) = 1/6$ is equal to the deficit $R(0) = -1/6$ while under the mechanism that maximizes revenue, welfare has fallen to $W(1) = 1/12$ while revenue has increased to only $R(1) = 1/24$.

4.4 Revenue-Efficiency Tradeoff in One-Sided versus Two-Sided Markets

Next we substantiate the notion that the tradeoff between revenue and efficiency is more salient in allocation problems with two-sided private information than in problems with private information on one side only. For the purpose of comparing the two allocation problems, we first need to determine the tradeoff between revenue and welfare in the one-sided problem. Without loss of generality, assume that sellers have no private information, but that each seller still draws his cost c_s independently from G. In order to induce seller s to sell, we therefore only have to pay him c_s. All other assumptions remain the same.

For any $\alpha \in [0,1]$, let $\mathbb{B}^{\text{one}}_\alpha(\mathbf{v},\mathbf{c})$ and $\mathbb{S}^{\text{one}}_\alpha(\mathbf{v},\mathbf{c})$ be the sets of buyers and sellers that would trade under efficiency if the true types of the buyers were $J^B(v_b,\alpha)$ for all $b \in \mathbb{B}$ and while the seller types are given by their true types c_s for all $s \in \mathbb{S}$. Denoting by $W(\alpha)^{\text{one}}$ and $R(\alpha)^{\text{one}}$ the expected welfare and revenue in the one-sided problem under a constrained efficient allocation rule, we have

$$W(\alpha)^{\text{one}} := E\left[\sum_{b \in \mathbb{B}^{\text{one}}_\alpha(\mathbf{v},\mathbf{c})} v_b - \sum_{s \in \mathbb{S}^{\text{one}}_\alpha(\mathbf{v},\mathbf{c})} c_s\right] \tag{28}$$

and

$$R(\alpha)^{\text{one}} := E\left[\sum_{b \in \mathbb{B}^{\text{one}}_\alpha(\mathbf{v},\mathbf{c})} J^B(v_b,1) - \sum_{s \in \mathbb{S}^{\text{one}}_\alpha(\mathbf{v},\mathbf{c})} c_s\right]. \tag{29}$$

As in the two-sided setup, revenue $R(\alpha)^{\text{one}}$ is increasing in α while welfare $W(\alpha)^{\text{one}}$ is decreasing in α for $\alpha \in [0,1]$. Observe then that when $\alpha = 0$, the sets of buyers and sellers that trade under efficiency are the same in the one-sided and two-sided problems because $J^S(c_s,0) = c_s$. Thus, $W(0)^{\text{one}} = W(0)$. In addition, when we consider the case with $\alpha = 1$, we have $J^S(c_s,1) > c_s$ for all $c_s > \underline{c}$, which implies that expected total welfare from the set of efficient trades by buyers with values $J^B(v_b, 1)$ and sellers with costs c_s is greater than from those with values $J^B(v_b, 1)$ and costs $J^S(c_s, 1)$. Thus, $W(1)^{\text{one}} > W(1)$. This establishes the following result.

Theorem 2 *The welfare lost when revenue is increased from its level under an ex post efficient allocation rule to its maximum level under any allocation rule is larger in environments with two-sided private information than in those with one-sided private information.*

To illustrate, reconsider the bilateral trade problem with F and G uniform on $[0, 1]$. We now have $W(\alpha)^{\text{one}} = \frac{1+2\alpha}{6(1+\alpha)^2}$ and $R(\alpha)^{\text{one}} = \frac{\alpha}{3(1+\alpha)^2}$. Thus, $W(0)^{\text{one}} = 1/6 = W(0)$ and $W(1)^{\text{one}} = 1/8 > 1/12 = W(1)$ and $R(0)^{\text{one}} = 0$ and $R(1)^{\text{one}} = 1/12$. Thus, in this illustrative example, the percentage of welfare lost, as a fraction of maximum welfare, as one goes from minimal to maximal revenue is 25% for the one-sided problem and 50% for the two-sided problem. This is depicted in Figure 2, where the efficient frontier is the thicker solid line for the two-sided problem and the thinner solid line for the one-sided problem.

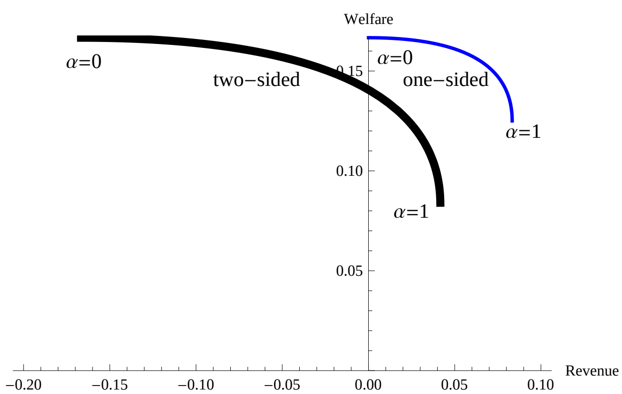

Figure 2: Efficient frontiers for one-sided and two-sided problems illustrating Theorem 2. The figure is drawn for the bilateral trade problem with uniform distributions.

According to Theorem 2, adjusting an allocation mechanism so as to increase expected revenue is more costly in terms of lost welfare with private information on both sides of the market than on just one. This is possibly an important insight for ongoing public debates as we discuss further in Section 6.

As mentioned, the expected revenue from a mechanism that places weight α on revenue, $R(\alpha)$, is increasing in α and satisfies $R(0) < 0$ under fairly general conditions. Moreover, under assumption (2), $R(1) > 0$ holds. This implies that there is a unique number $\alpha^* \in (0, 1)$ such that $R(\alpha^*) = 0$ holds. Because $W(\alpha)$ is strictly decreasing in α, it follows that a mechanism that implements the α^*-allocation rule is a second-best mechanism in the sense of maximizing ex ante expected surplus subject to budget balance (in expectation) (and incentive compatibility and individual rationality). Myerson and Satterthwaite (1983) derive such a second-best mechanism for the bilateral trade problem and notice that when values and costs are drawn from a common uniform distribution, it can be implemented by the linear equilibrium of the double auction of Chatterjee and Samuelson (1983).[48]

5 Synthesis for the Future

In this section we discuss a number of open issues of importance for the design of centralized two-sided mechanisms and place them in the context of the existing literature.

5.1 Deficit-free Mechanisms for Practical Implementation

The analysis and discussion above lead naturally to the question of whether there are practical mechanisms that implement “almost” efficient allocation rules that do not run a deficit. To address this question, we begin by reviewing the experimental literature on continuous-time double auctions that have guided the design of multiple real world mechanisms over the last two decades. Next we illustrate McAfee's (1992) detail-free dominant strategy mechanisms for centralized two-sided markets with unit demands and unit supplies. Then we describe recent proposals for detail-free dominant-strategy mechanisms in environments with multi-unit demands and supplies and interdependent values and explain briefly why extensions to the setup of multi-unit traders are challenging.

Continuous-Time Double Auctions Starting with the work by Smith (1962) and Smith (1964), a large experimental literature has documented the remarkably efficient performance of the continuous-time double-auction, which is an open auction format similar to the one used, say, in the New York Stock Exchange. In the continuous-time double auction, trade occurs continuously over a fixed time interval and both buyers and sellers can submit bids and asks to a centralized exchange. Trade in this mechanism occurs any time a buyer's bid is above a seller's

[48] On the existence and efficiency of equilibria in the double auction, see Satterthwaite and Williams (1989). More generally, Loertscher and Niedermayer (2013) observe that with possibly many buyers and one seller, any α-allocation rule can be implemented with a fee-setting mechanism, according to which the seller sets a (reserve) price in a second-price auction and, upon trade at the transaction price t, the market maker receives the fee $t - E[J_\alpha^{S^{-1}}(J^B(v,\alpha))|v \geq t]$, where $J_\alpha^{S^{-1}}(\cdot)$ is the inverse of $J^S(c,\alpha)$ with respect to c. Lastly, Gresik and Satterthwaite (1989) analyze the convergence of welfare toward full efficiency under second-best mechanisms as the markets become large. For related convergence results, see also Rustichini, Satterthwaite, and Williams (1994) and Cripps and Swinkels (2006) for double auctions and Tatur (2005) for VCG mechanisms with participation fees. Satterthwaite and Williams (2002) provide an analysis of worst-case asymptotic optimality.

ask. In terms of efficiency and convergence to the competitive equilibrium, this format performs as well as or better than other auction formats such as sealed bid-offer auctions (Smith et al. (1982)), uniform price double auctions (McCabe, Rassenti, and Smith (1993)), posted-offer markets (Walker and Williams (1988)), and multiple-call markets (Friedman (1993); Cason and Friedman (2008)). The success of two-sided markets in laboratory settings has led to a number of centralized 'smart' markets that facilitate exchange in homogeneous goods markets with network externalities such as electricity (Rassenti, Smith, and Wilson (2003); Wilson, Rassenti, and Smith (2003)), gas (McCabe, Rassenti, and Smith (1989a, 1990)), and water (Murphy et al. (2000); Murphy et al. (2009)).

While important in practice and successful in the experimental lab, a shortcoming of continuous-time double-auctions is that that equilibrium behaviour is difficult to understand. As noted by Satterthwaite and Williams (2002), this lack of theoretical understanding may have been an impediment to progress in two-sided market design. In what follows, we describe dominant-strategy mechanisms that may provide a basis for further theoretical and eventually practical developments.

Detail-Free Dominant Strategy Mechanisms McAfee (1992) introduces the following dominant strategy double auction for homogenous goods with single unit demand and supply. All buyers and sellers submit bids simultaneously.[49] Let $v_{(h)}$ denote the h-th highest bid submitted by a buyer, $c_{(h)}$ the h-th lowest bid submitted by a seller, with $v_{(B+1)}$ and $c_{(S+1)}$ defined as $v_{(B+1)} = \underline{v}$ and $c_{(S+1)} = \overline{c}$, and define $p_0 := (v_{(k+1)} + c_{(k+1)})/2$, where k denotes the efficient quantity defined with respect to the submitted bids $(\mathbf{v}, \mathbf{c})$, that is, k is the largest integer such that $v_{(k)} \geq c_{(k)}$. The allocation and pricing rule under McAfee's mechanism is as follows. If $p_0 \in [c_{(k)}, v_{(k)}]$, then the efficient quantity k is traded at the uniform price p_0. If $p_0 \notin [c_{(k)}, v_{(k)}]$, the quantity traded is $k - 1$, and all buyers who trade pay $v_{(k)}$ and all sellers who trade are paid $c_{(k)}$. Agents who do not trade make and receive no payments. Because the mechanism is budget balanced in the first case and does not run a deficit in the second case, it never runs a deficit. Further, agents have a dominant strategy to bid their type by the second-price nature of the mechanism—no agent can affect the price that he or she pays or gets, given that he or she trades.

[49] Alternative dominant strategy double auctions were introduced by Yoon (2001) and Tatur (2005) for setups with unit traders and by Yoon (2008) for the case of multi-unit sellers and buyers. In contrast to the mechanisms we discuss here, these mechanisms are not detail-free in the sense of Wilson (1987).

A lesson from the analysis above is that under fairly general conditions some surplus has to be sacrificed in order to avoid deficits in markets with two-sided private information. The α^*-mechanisms, which were first analyzed by Myerson and Satterthwaite (1983) and Gresik and Satterthwaite (1989), achieve this by inducing trade only for the traders that belong to the efficient sets defined with respect to $J^B(v_b, \alpha^*)$ and $J^S(c_s, \alpha^*)$. However, these concepts, and thus the α^*-mechanisms, are Bayesian notions that depend on the fine details of the design problem at hand. They thus violate the robustness requirement often associated with the Wilson Doctrine, which postulates that in order to be practical, mechanisms should be free of such details (Wilson, 1987). The Wilson Doctrine is particularly relevant when the design problem at hand is a one-shot allocation mechanism in which learning and convergence to the true parameters cannot occur. McAfee's mechanism, in contrast, sacrifices efficiency by preventing trade by the least efficient pair of traders, if it prevents any trade at all. Because this pair is well-defined for any submitted bids regardless of the process that determines agents' types, the mechanism is in line with the Wilson Doctrine.[50]

An important question that McAfee (1992) leaves open is how or to what extent the mechanism can be generalized to the empirically often more relevant case where buyers have multi-unit demands and sellers have multi-unit supplies. Such a generalization faces two challenges: The efficient trades that must be foregone need to be identified, and demand and supply of those agents who trade have to be balanced.[51] Loertscher and Mezzetti (2014) show that both challenges can be overcome in a setting where goods are substitutes by the following mechanism. Let all agents report their marginal values and costs for all units. Temporarily, restrict attention to the reports for the first units only, that is, the highest marginal values reported by buyers and the lowest marginal costs reported by sellers, and determine the efficient quantity with respect to the reports on the first units. Relative to this quantity and the reports on first units, one can then determine prices for buyers and sellers in the same way as is done by McAfee (1992). In contrast to McAfee, however, these prices are not necessarily the ultimate transaction prices. They serve as reserve prices in a two-sided VCG-auction in which the reports on all units by all buyers and sellers are used. The short side of the market trades at

[50]However, the price p_0 depends on the bounds of the distribution in case $k = \min\{B, S\}$. This is for example the case in the bilateral trade problem whenever $v_b > (\underline{v} + \overline{c})/2 > c_s$. In this way, the mechanism depends on some details of the design problem although it is independent of any other assumptions about distributions. It should also be noted that in exactly the same vein the VCG-mechanism is not detail free (in the two-sided setup) despite being independent of distributional assumptions.

[51]Observe that the latter constraint is automatically satisfied in the unit case.

the reserve and the long side trades at endogenously determined unit prices. These are higher than the reserve price if buyers are on the long side and smaller than the reserve price if sellers are on the long side.

Kojima and Yamashita (2014) propose an alternative mechanism for two-sided environments, allowing for interdependent values, which they call the group-wise price mechanism. Essentially, the mechanism divides the market into a number of groups (or submarkets) and then uses group-specific prices. Trade in each submarket is restricted to be between buyers and sellers belonging to this submarket, but the reserve price at which these trades occur are determined by the reports from some other submarket. Market clearance is assured by using generalized VCG-auctions.

At a fundamental level, a key challenge to the design of detail-free and deficit-free dominant strategy mechanisms in environments with multi-unit traders and private information on both sides of the market appears to be the following. While it is often not difficult to use the reports by the agents on one side of the market to determine prices for agents on the other side in an incentive-compatible manner, the requirement that the market must clear then almost inevitably generates a link between the prices that agents on one side of the market face and their own reports. This is a major challenge for maintaining the dominant strategy property. In McAfee's (1992) setup with unit traders, this challenge can be overcome because an agent—say, a buyer—can only affect the price that buyers face by becoming inactive or by paying a price above her value. With multi-unit traders, this is no longer necessarily the case and so demand (and supply) reduction becomes a problem.[52]

5.2 Combined One-Sided and Centralized Two-Sided Market Design

In many real-world applications, the designer or market maker participates as an organizer and seller in the primary market and may act as the organizer of the secondary market. Consequently, the option of combining one-sided and centralized two-sided markets is a practical possibility. In the following, we argue that the designer has the potential to decrease the expected deficit of the resale market and increase efficiency by linking these two activities and by releasing additional units for sale within the context of a centralized two-sided market.[53]

[52] In Loertscher and Mezzetti (2014) this problem is solved by making an agent's activity (but not his quantity traded) a function only of the reports on the first units. Kojima and Yamashita (2014) address the problem by using reports from agents in another (pre-determined) submarket to set the reserve prices that determine an agent's activity.

[53] Kwerel and Williams (2002) propose a two-sided auction for spectrum licenses that includes unassigned spectrum held by the FCC.

The main idea is simple and based on the insight that efficient mechanisms that run a surplus exist for one-sided markets but not for centralized two-sided markets. Therefore, by combining the two, it may be possible to achieve efficiency in both markets without running a deficit in expectation.

In this section, we again stay within our basic setup where buyers have single unit demand and sellers have single unit supply, but we limit attention to the case of two buyers and one seller. In addition, we extend the model to allow the mechanism designer to also have a unit that it can sell, at a cost of zero to itself.

Consider a situation with $B = 2$ buyers who draw their values for a homogenous good independently from the distribution F with support $[0, 1]$ and density f and one private seller whose cost c is drawn from the distribution G with the same support and density g. The mechanism designer owns one unit of the good, which he values at 0.

We start by evaluating the revenue of the VCG mechanism when the designer's additional unit is not allocated. We then show how the expected revenue from the VCG mechanism improves when the designers additional unit is released. We then discuss how combined mechanisms might be designed that never run a deficit and discuss the welfare and revenue tradeoff in combined one-sided and two-sided auctions relative to pure one-sided auctions.

Because of its second-pricing nature, it is a weakly dominant strategy for each individual to report truthfully. Without the release of the designer's unit, there are three potential orders of reports: (i) $v_{(2)} < v_{(1)} < c$, (ii) $v_{(2)} < c < v_{(1)}$, and (iii) $c < v_{(2)} < v_{(1)}$. In case (i), no trade occurs; in case (ii), the highest-valued buyer and the private seller trade at a net deficit of $v_{(1)} - c$; and in case (iii), the highest-valued buyer and the seller trade at a net deficit of $v_{(2)} - c$. Thus, without the additional unit, there is a deficit in all cases with trade. When the designer's unit is released, by contrast, the mechanism yields a surplus of $v_{(2)}$ in cases (i) and (ii) due to the higher-valued buyer purchasing the additional unit and paying the opportunity cost of the lower-valued buyer. In case (iii), both units are traded and both buyers pay c while the seller receives $v_{(2)}$. In all three cases, the revenue from the auction with the additional supply is strictly higher. However, it is not necessarily positive even with the release of the designer's additional unit. Next, we briefly derive conditions under which expected revenue is positive when the designer's unit is added.

The expected revenue from the mechanism with the additional unit can be written as

$$\int_0^1 \int_0^v (2c - v) g(c) f_{2,2}(v) dc dv + \int_0^1 v f_{2,2}(v)(1 - G(v)) dv \quad = \quad \int_0^1 \left(v - 2 \int_0^v G(c) dc \right) f_{2,2}(v) dv,$$

where $f_{2,2}$ is the density of the second-highest of two random variables drawn from distribution F and where the equality uses integration by parts. A simple geometric argument reveals that $2\int_0^v G(c)dc < v$ for any distribution G that first-order stochastically dominates the uniform distribution.[54] Thus, a sufficient condition for expected revenue to be positive is that $G(c) \leq c$ for all $c \in [0, 1]$. However, this condition is clearly only sufficient.[55] The extent to which insights on combined market design extend to more general setups is an open question that seems relevant for the literature on and the practice of two-sided market design.

In principle, the mechanism designer could match the efficiency and revenue characteristics of the combined VCG mechanism by running an initial auction that sells the additional unit and then running a two-sided mechanism. However, separate one-sided and two-sided markets may be problematic in practice if the seller cannot commit to running a deficit in the two-sided market despite positive revenue in the one-sided market. In addition, if the winner in the one-sided market can participate as a seller in the two-sided market, then other "collusive" equilibria exist in the one-sided market with low revenue, as described by Garratt, Tröger, and Zheng (2009).

In practice, a designer might not only face the constraint that there must be no deficit in expectation, but also that there must be no deficit ex post. In the following, we briefly describe a connected, detail-free mechanism (i.e., a mechanism that does not rely on knowledge of the distributions from which bidders draw their values and costs) that achieves this objective.

Within our model in which buyer and sellers have single unit demand and supply, allow the government to also be a source of supply. Let $K \geq 1$ be the number of units owned by the government seller who values them at 0. There are M private sellers and N buyers with $N \geq K + M$ for simplicity. Because usual, let $v_{N+1} = \underline{c}$ and $c_{M+1} = \overline{v}$, where $\underline{c}$ is the lowest possible cost of a private seller (typically 0) and $\overline{v}$ is the highest possible valuation of a buyer.

Letting $k \in \{K, \ldots, K + M\}$ be the quantity traded and $v_{(i)}$ be the ith highest bid of a buyer and $c_{(j)}$ be the jth lowest bid by a private seller with $c_{(0)} = 0$, the private sellers who trade are paid the price $p_S(k) := \min\{c_{(k+1-K)}, v_{(k)}\}$ and the buyers who trade pay $p_B(k) := \max\{v_{(k+1)}, c_{(k-K)}\}$. This induces dominant strategies for the usual reasons for any given k. The net revenue that accrues to the mechanism designer as a function of k is

[54] The argument is this: For G uniform, $\int_0^v G(c)dc$ is equal to the symmetric triangle with length and height v, whose size is thus $v^2/2 < v$. For any distribution that stochastically dominates the uniform, $\int_0^v G(c)dc$ will be smaller than this triangle.

[55] Assuming, for example, that F is uniform and $G(c) = c^\sigma$ with $\sigma > 0$, expected revenue can be shown to be positive for any $\sigma > 0.44$.

$R(k) := kp_B(k) - (k-K)p_S(k)$. Choosing k such that $R(k) \geq 0 > R(k+1)$ guarantees a nonnegative revenue.[56]

The assumption that the designer's units have zero costs may be appropriate in some instances but may seem stark in others. To make a comparison between one-sided and two-sided setups meaningful, assume that the designer draws his opportunity costs for selling each unit he owns independently from the same distribution G as the private sellers. Let $K \in \{0, .., S\}$ be the number of units owned by the designer, and assume that the number of private sellers is $S-K$. Let $W^{\text{con}}(\alpha, K)$ be expected welfare under a constrained efficient allocation rule in a connected setup where the designer owns K units with $S-K$ private sellers.[57] The following simple but possibly valuable result is then an immediate corollary to Theorem 2.

Corollary 1 *The welfare loss from revenue maximization decreases in K. That is, $W^{con}(0,K) - W^{con}(1,K)$ is decreasing in K.*

5.3 Understanding Incentives for Market Making

Government intervention in market making in two-sided environments is, in some sense, a more fundamental interference with the actions and plans of private agents than is, say, auction design by governments who sell or procure assets. In such one-sided environments, the market designer is a party to the transaction, whereas in two-sided environments this is typically not the case.

Clearly, the debate about whether government intervention in two-sided environments is warranted will continue. In order to be insightful, ultimately an equilibrium theory of where and when (constrained) efficient centralized exchanges emerge endogenously will be required to discipline the discussion about whether in a given situation the incentives of private agents to create exchanges are socially suboptimal, resulting in what could be called *market failures in market making*. Despite the emergence of a literature on market making and the microstructure of markets over the last two decades or so and of the still burgeoning literature on two-sided platforms and platform competition, it appears that such a theory is still missing.

For example, the early work by Stahl (1988) and Yanelle (1989) focuses on competition between middlemen who allow buyers and sellers to trade, but does not address the outcome

[56] Efficiency could be further increased, at the cost of additional complexity, if one changed the pricing rule to the following element in the spirit of McAfee (1992). If the efficient quantity is k and $v_{(k+1)} \geq c_{(k)}$ and $c_{(k+1)} \leq v_{(k)}$, let the price be $p = p_B = p_S = (v_{(k+1)} + c_{(k+1)})/2$. (One could still do a little better than that because the mean is somewhat arbitrary).

[57] When the quantity traded is N, a private seller s now sells if and only if his cost c_s is such that $J^S(\alpha, c_s)$ is smaller than the Nth smallest element of the K cost draws of the designer and of the $J^S(\alpha, c_j)$ of his $S-K-1$ competitors j.

absent intermediaries. In the models of Rubinstein and Wolinsky (1987), Yavas (1992), Gehrig (1993), Spulber (1996), Spulber (2002), Rust and Hall (2003), Duffie, Garlenau, and Pedersen (2005), Loertscher (2007) and Neeman and Vulkan (2010), the market makers' profit is constrained by the agents' outside opportunity of trading outside the market makers' exchange, but the questions under what conditions it is socially (and privately) optimal to set up an organized exchanged are not addressed.[58]

The literature on two-sided platforms has primarily focused on the platforms "chicken-and-egg"-problem of bringing both sides of the market on board and on the effects of competition between such platforms as a function of the pricing instruments available to the them; see, for example, Caillaud and Jullien (2001, 2003), Rochet and Tirole (2002, 2003) and Armstrong (2006).[59] Optimal price regulation has been an important policy addressed within this literature, but to the best of our knowledge the conditions under which a social planner would want more (or less) platforms to emerge than operate in equilibrium has not been analyzed.

The less efficient decentralized markets are, the larger will be society's interest in having a centralized (constrained) efficient market maker. However, this need not mean that the tendency for market failure is strongest when the decentralized market is most inefficient. Typically a market maker's profit is larger the greater is this inefficiency. Therefore, if a decentralized market is highly inefficient, then there will, all else equal, be strong incentives for a market maker to enter.

5.4 Additional Issues in Two-Sided Market Design

In this section we discuss issues of collusion, complexity, and information sharing that have contributed to the designs of various one-sided auction formats, as well as privacy and costs of delay. We highlight potential differences in the way these issues seem likely to impact two-sided auction design, and we discuss open questions for future work.

5.4.1 Collusion

In thin two-sided markets, collusion across buyer and seller pairs may be quite lucrative to the pair and quite costly to the designer. Consider, for example, the bilateral trade problem in an independent private values environment with the value and cost drawn from the uniform distribution on $[0, 1]$. In a VCG mechanism, the designer's expected payment is $1/6$ if the buyer

[58] Spulber (1999) provides an early survey of this literature.

[59] Rochet and Tirole (2006) and Rysman (2009) provide surveys of this literature.

and the seller bid truthfully. If the two parties collude by having the buyer always submit a value of 1 and the seller always submit a bid of 0, however, the object is always traded and the designer pays 1 regardless of the realized types.[60]

Because buyer and seller pairs may have strong incentives to collude, concerns for collusion in two-sided markets have understandably been raised in the literature. For example, Rothkopf (2007, p.192) raises the issue of "conspiracies in two-sided markets between bidders offering to sell and those offering to buy," while Hobbs, Rothkopf, and O'Neill (2000) discusses how to construct such conspiracies in two-sided electricity markets. The LIBOR scandal, where reports by potential buyer and seller trading pairs guided the prices paid in transactions with third parties, demonstrates that such concerns are likely important in practice.[61]

It is well-known that the potential for collusion between bidders in one-sided setups depends, among other things, on the mechanism used (see e.g. Marshall and Marx, 2007, 2009). The same will be true for collusion between buyers and sellers in two-sided setups and for the effect that buyer-seller collusion has on the efficiency of the resulting allocation.[62]

Interestingly, once one considers mechanisms that account for the revenue-efficiency tradeoff inherent in two-sided setups by sacrificing some efficiency in order to avoid deficits, profitable collusion between buyers and sellers may be efficiency enhancing. This is, for example, the case for McAfee's (1992) mechanism, which appears to be relatively robust to buyer and seller collusion. Recall from our discussion in Section 5.1 that when $p_0 = (v_{(k+1)} + c_{(k+1)})/2 \notin [c_{(k)}, v_{(k)}]$, this mechanism trades $k-1$ units, the buyer pays $v_{(k)}$, and the seller receives $c_{(k)}$. However, when $p_0 \in [c_{(k)}, v_{(k)}]$, the efficient quantity k is traded at the uniform price $p_0 = (v_{(k+1)} + c_{(k+1)})/2$. In cases where $c_{(k+1)} \leq v_{(k)}$ but $p_0 < c_{(k)}$, the kth seller and the $k+1$ (or higher) buyer have an incentive to form a coalition and have the buyer submit a bid equal to $c_{(k+1)}$. This bid will lead to the kth unit being traded at a trade price that is lower

[60]This example is particularly severe due to the use of the VCG mechanism, where truth-telling is only a weak best response. We use it only as an example to highlight the additional incentives for collusion that may be possible when deficits are paid by a third party. See Ausubel and Milgrom (2006) for a broader discussion of this issue.

[61]On the construction and use of the LIBOR and the LIBOR scandal, see Hou and Skeie (2014). The industrial organization and contracting literature has also been concerned with collusion when third parties play the role of budget-balancers. See Eswaran and Kotwal (1984) and Holmstrom (1982) for early papers in this literature.

[62]There is a small experimental literature that studies how market institutions impact the ability of bidders to collude. The double oral auction typically used in the experimental literature, for instance, does not appear to be particularly susceptible to collusion relative to posted offer formats (Isaac and Plott (1981); Clauser and Plott (1992)). In one-sided settings with buyers, ascending bid auctions and first-price clock auctions appear to be sensitive to tacit collusion, while descending clock auctions do not appear to have this feature (Li and Plott (2009)). Experimental work also suggests that sealed-bid markets can be vulnerable to collusion if certain types of communication are allowed (Isaac, Ramey, and Williams (1984); Isaac and Walker (1985); Saijo, Une, and Yamaguchi (1996); Artale (1997); Kwasnica (1998)).

for all buyers and higher for all sellers. Likewise, in cases where $v_{(k+1)} \geq c_{(k)}$ but $p_0 > v_{(k)}$, the $k+1$-st (or higher) seller and the k-th buyer have an incentive to form a coalition and have the buyer submit a bid equal to $v_{(k+1)}$. These coalitions lead to higher surplus for all trading parties at the same time that they increase efficiency. Thus, collusion between buyers and sellers under this almost-efficient mechanism can work in favor of increasing efficiency.[63]

The transition from one-sided mechanisms to two-sided mechanisms may also have implications for the ability of one side of the market to collude or tacitly collude with one another. As noted by Milgrom (2004, Section 7.2), for instance, a key concern in the design of uniform price auctions in settings where bidders can demand multiple units is the possibility of equilibrium prices that are far from the competitive levels. In cases where supply is known and inelastic, low-price equilibria exist for a variety of both sealed-bid and clock auctions where the revenue is close to the seller's reserve. When supply is elastic, by contrast, the worst auction outcome resembles the results of Cournot competition among buyers. Green and Newbery (1992) and Klemperer and Meyer (1989) find that prices lie between prices in the Cournot equilibrium and the competitive equilibrium when supply is elastic and uncertain. In their models, the competitive equilibrium is the only equilibrium that exists when uncertainty regarding supply is sufficiently large. Thus, in a two-sided setting where uncertainty over the sellers' values generates both uncertainty and elastic supply, there are reasons to believe that some non-competitive equilibria may disappear.[64]

As discussed in Section 2, designers of one-sided auctions have taken steps to limit the ability of bidders to collude. Similar steps will be of value in the design of a centralized market with privately informed buyers and sellers.[65] A set of important open questions concerns the extent to which the potential for collusion in two-sided mechanisms differs from what we know for their one-sided counterparts.[66]

[63]This is in contrast to the VCG mechanism noted above. In the homogeneous goods VCG mechanism, the inframarginal buyer and seller pair always have an incentive to collude. This collusion can be deficit increasing and efficiency decreasing.

[64]See Rassenti, Smith, and Wilson (2003) for experimental evidence of these effects as they relate to electricity markets.

[65]The incentive auction proposal of Milgrom et al. (2012) includes an information policy that would limit the ability of bidders to monitor the behavior of other bidders by not revealing bidding details during the auction.

[66]The concern for collusion in dynamic auctions stems from the potential for bidders to condition future actions on past events. This argument was first put forth in Mead (1967) and first analyzed formally in Robinson (1985). Theoretical arguments supporting the relative susceptibility of dynamic versus sealed-bid auction formats can be found in Klemperer (2002b), Brusco and Lopomo (2002), Marshall and Marx (2007), and Marshall and Marx (2009). Empirical evidence that collusion may be easier to sustain in dynamic auction formats can be found in Athey, Levin, and Seira (2011). Experimental evidence directly related to this issue can be found in Hu, Offerman, and Onderstal (2011) and Hinloopen and Onderstal (2010).

5.4.2 Complexity of Multi-object Auctions with Complements and Substitutes

In many of the settings for which two-sided markets are likely to be designed, objects will be heterogeneous and individuals will perceive some objects as complementary. A major concern in the design of mechanisms in such complex settings is whether potential bidders are able to understand the mechanism and whether bidders act in these environments in a way predicted by theory and intended by the designer.[67] Such concerns may be heightened in two-sided settings where complementarities between buyers and sellers and uncertainty over available supply may lead to greater complexity in analyzing potential bids and in developing optimal strategies.[68]

While auction complexity is a nascent field, one clearly emerging pattern is that bidders fail to analyze the full set of potential bids even in simplified combinatorial settings. In Kagel, Lien, and Milgrom (2010), for example, bidders participated in a Porter et al. (2003) style combinatorial clock auction that allows for multiple package bids in each period and use an 'XOR' bidding language to allocate objects. Despite having many packages that were potentially profitable, bidders typically concentrated their bids on one or two packages each round that maximized current period profit. When these packages did not correspond to packages that were predicted by theory to be "efficiency relevant," overall revenue and efficiency fell.[69] A similar result is found in Scheffel, Ziegler, and Bichler (2012) and Bichler and Shabalin (2013), who study the clock-proxy auction of Ausubel, Cramton, and Milgrom (2006). In the clock-proxy auction, bidders are allowed to submit a single bid in each round of the clock round, followed by a supplementary sealed-bid phase, where bidders can submit bids over all packages. The authors find that in environments with a large number of packages, bidders analyze and bid on only a small set of supplementary bids. This leads to a decrease in overall efficiency.[70]

In two-sided settings, where there is an inherent complementarity between buyers and sellers, bidding on only a subset of profitable packages may have profound effects on the efficiency

[67] Concerns over complexity played a major role in the design of the original FCC auction. See Plott (1997) and the introduction by Kwerel in Milgrom (2004) for detailed discussions.

[68] As stated by FCC Commissioner McDowell in Congressional Hearings on the incentive auction, for instance, "Quite simply, the incentive auctions will be the most complex in world history and the entire process may take the greater part of a decade. I urge the Commission to work in a deliberate and transparent manner, with an eye toward simplicity, humility and restraint." ("Keeping the New Broadband Spectrum Law on Track" (U.S. House Energy and Commerce Committee, 12 Dec. 2012), pp.2–3)

[69] Brunner, Goeree, Holt, and Ledyard (2010) also find that individuals do not bid on all packages in a simultaneous multi-round format that uses an 'XOR' bid language and allows for package bidding (SMRPB). They find that this design hurts efficiency in environments where complementarities are low.

[70] Testing the VCG mechanism, Chen and Takeuchi (2010) and Scheffel, Ziegler, and Bichler (2012) also find that profitable packages are not bid on. Similar to the results for second-price sealed-bid auctions, they also find underbidding and overbidding across packages.

of the auction. For example, if sellers have multiple goods and are willing to sell all of them or none of them, an incomplete set of bids by buyers on some goods may lead sellers to retain others. This inherent threshold problem is not present in most one-sided problems where supply is typically fixed.

In many of the common iterative auction formats, straightforward bidding is a Nash equilibrium when goods are substitutes. Thus, the complexity of bidding in an auction can be related to the complementarities that exist across goods. In one-sided settings, potential complementarities can often be eliminated through careful prepackaging of objects.[71] However, in some two-sided market settings, the ability for the designer to prepackage may be limited.[72] Uncertainty over supply and the potential for very different goods to be sold may make it difficult for the designer to eliminate complementarities. Designs that can accommodate complementarities may therefore be of great importance with two-sided private information.[73]

5.4.3 Information

The literature related to the "linkage principle" first identified by Milgrom and Weber (1982) and also referred to as the "publicity effect" in Milgrom (2004), asserts that when the mechanism designer of a one-sided mechanism possesses private information that is affiliated with the signals of all other agents,[74] expected revenues are increased when the designer commits to a policy of always revealing his private information. The logic of this result stems from the fact that winners, when bidding naively, overestimate the other bidders' signals and suffer from the winner's curse. Rational bidders will reduce their bids in order to avoid the problem of the winner's curse. New information that is affiliated with the other bidders' values reduces the expected magnitude of overestimation from a winning bid and increases every bidder's bid on average.

In one-sided settings with symmetrically informed bidders, the linkage principle implies that

[71] For example, in the sale of 700 MHz spectrum, most countries sold licenses in pairs. These paired licenses were ideal for LTE-based 4G networks, which used one frequency for transmitting information and another license for transmitting. Most countries also sell licenses as generic lots and assign specific licenses only at the end of the auction to avoid frequency fragmentation.

[72] The proposed rules for the broadcast incentive auction propose selling all spectrum as generic lots and then having the FCC optimize the resulting spectrum for the bidders after the auction.

[73] Milgrom et al.'s (2012) incentive auction proposal is attentive to the issue of complementarities and substitutability among licenses. Their proposal does not use combinatorial bidding, but their proposed intra-round bidding would be evaluated as a package, allowing bidders to use that mechanism to mitigate exposure risk.

[74] "Loosely, two signals are affiliated if a higher value of one signal makes a higher value of the other signal more likely, *and* this is true on every subspace of the variables' domain. Thus, affiliation is stronger than correlation, which is a global summary statistic; affiliation can be thought of as requiring local positive correlation everywhere." (Klemperer, 1999, p.254).

formats such as the ascending-bid auction where the incremental bids of buyers are observable will yield higher revenue than a sealed-bid format.[75] This powerful result has been one of the main arguments for ascending bid formats in the design of spectrum auctions and has led to a push for full information revelation in a number of auction settings.[76,77]

In symmetric two-sided settings, the assumptions of affiliation and interdependent appears to be enough to ensure that the bids of both buyers and sellers increase on average when information is revealed.[78] However, because the expected revenue generated in a two-sided mechanism is determined by the difference in bids across sets of buyers and sellers, the assumptions underlying the linkage principle are not necessarily sufficient to ensure that revenue increases with the release of information. Research into identifying necessary conditions under which information unambiguously increases revenue is likely to be valuable moving forward. Additionally, because two-sided problems will naturally be in places where there are asymmetries among bidders, continued research into the linkage principle in asymmetric environments is likely to be important.[79]

5.4.4 Emerging issues

Privacy and Costs of Delay Additional issues arise in the design of two-sided markets that play less of a role in their one-sided counterparts. For example, delay in one side of a two-sided market can affect costs and participation on the other side,[80] and additional privacy concerns

[75] More formally, if bidders' expected values are increasing in their types and their types are affiliated (see, e.g., Milgrom, 2004, Section 5.4.1), then for each type of bidder, the conditional expected price in the ascending-bid auction, given that the type wins, is higher than the corresponding bid at the first-price auction (see, e.g., Milgrom, 2004, Theorems 5.4.14 and 5.4.17). Furthermore, under additional conditions, the equilibrium of the ascending-bid auction is efficient (see, e.g., Milgrom, 2004, Theorem 5.4.12 for the definition of the relevant equilibrium bidding strategies and Theorem 5.4.13 on efficiency).

[76] In the words of Evan Kwerel, senior economist in the Office of Plans and Policy at the FCC, "In the end, the FCC chose an ascending bid mechanism, largely because we believed that providing bidders with more information would likely increase efficiency and, as shown by Milgrom and Weber (1982), mitigate the winner's curse." (Introduction by Kwerel in Milgrom, 2004, p.xvii)

[77] Research from experiments suggest that open formats also facilitate learning and price discovery. See Kwasnica and Sherstyuk (2013) for a review of this literature in relation to multi-unit auctions. See the reviews of Kagel and Roth (1995) and Kagel and Levin (2013) for a survey of this literature as it relates to English auctions and second-price sealed-bid auctions.

[78] The alternative version of the linkage principal constructed in Krishna and Morgan (1997) and used in Krishna (2002) can be extended directly to buyers and sellers in a two-sided environment to establish this result.

[79] As shown in Krishna (2002), the linkage principle does not hold with asymmetric bidders. In these asymmetric environments, revealing information can change the order of bidders' values and influence the allocation. See Milgrom (2004) for a simple example and Board (2009) for a more general discussion. The linkage principle also does not hold in multi-unit auctions (Perry and Reny, 1999).

[80] Milgrom et al. (2012) are attentive to the significance of costs of delay in their proposed incentive auction design: "A faster Forward Auction is valuable because the outcome of the Reverse Auction cannot be determined until the nearly completed Forward Auction lets the FCC decide how much it can afford to pay to clear spectrum. Long delays in the Forward Auction could raise costs and discourage participation in the Reverse Auction."

can arise if, for example, the two-sided mechanism calls for additional contingent bids to be collected from buyers to address uncertainty regarding the level of supply from sellers.

Market Thickness and Market Clearing The driving idea behind designing centralized exchanges is that larger markets are typically more efficient than smaller ones. This notion is formalized, for example, by Gresik and Satterthwaite (1989), who study the rate of convergence to full efficiency of a market under the Bayesian second-best mechanism, and by Rustichini, Satterthwaite, and Williams (1994) and Cripps and Swinkels (2006), who study large double-auctions and the speed of convergence to efficiency. Tatur (2005) introduces a double-auction with transaction fees and characterizes the asymptotic frontier between efficiency and revenue. His results imply that inefficiency per trader vanishes as markets become large.[81]

In a two-sided setup in which the designer is not a party to any of the transactions, the question of how frequently the designer should run the secondary market becomes salient, with the tradeoff being between opportunity costs of delay when letting buyers and sellers accumulate without clearing the market and the efficiency loss associated with smaller but more frequently cleared markets.

Joint Development of Technology, Property Rights, and Allocation Mechanisms Market designers, as is standard in much of economics, take technology and the legal institutions, such as the property right associated with a spectrum license, as a given. Their aim is then to design market mechanisms that induce a desirable allocation of these legal entitlements, given the existing technology. However, the development and deployment of new technology also depends on the legal arrangements and the institutions of exchange available. To take a concrete example, an alternative to allocating property rights of spectrum to a given user, one could allow firms to bid for the right to use that spectrum in real time.

Clearly, there is a degree of speculation underlying the argument that one should account for the effects of institutional choice on new technologies. However, such speculation is to

(Milgrom et al., 2012, p.4) To address this, Milgrom et al. (2012) propose to modify the FCC's SMR design to have generic licenses, to have a clock auction format, and to use intra-round bidding, which has the potential to increase the speed of the auction by allowing the auctioneer to use larger price increments without risk of "overshooting" the market clearing price.

[81]A cautionary message concerning the desirability of large markets emerges from Dang (2013). He analyzes noise trading and costly information acquisition in a common value environment without private information (beyond the information about the common value some traders may acquire endogenously). He shows that a pure strategy equilibrium that is efficient exists for large information costs and a small number of traders. In contrast, with a large number of traders, there is no pure strategy equilibrium and thus no efficient equilibrium.

some extent inevitable when it comes to the development of new technologies and does not invalidate the argument itself. Moreover, alternative technologies may already exist and be ready to be deployed, but may not be pursued if the existing legal and economic institutions do not make deployment worthwhile (or possible).[82] The research agenda and the political process required to make progress along these lines is, of course, somewhat vague and unclear. Nonetheless, it seems that considerable value could be added by thinking systematically about the larger "design" problem that considers legal and economic arrangements simultaneously with technological innovations and applications.

6 Conclusion

The existing economics literature and the history of spectrum auctions that have established foundations that can be employed to guide the design of centralized two-sided markets for environments where both buyers and sellers have private information. Because impossibility results in the literature imply zero or negative expected revenue from a fully efficient two-sided mechanism, the designers of two-sided markets must address the tradeoff between revenue and efficiency. The non-distortionary lump-sum transfers that would be required to finance ex post efficient trade do not seem feasible in the real world.[83] This raises the question of what (Bayesian) mechanism maximizes welfare subject to budget balance. According to Theorem 2, adjusting an allocation mechanism so as to increase expected revenue is more costly in terms of lost welfare with private information on both sides of the market than on just one.

These insights are relevant for policy, such as the debate over the extent to which the FCC's incentive auction should be designed with revenue maximization in mind.[84] Historically one-sided spectrum license auctions have raised large amounts of money for the U.S. Treasury

[82]For example, although technologies such as cognitive radio exist that allow devices to operate in multiple spectrum bands, the use of this technology has been constrained by the way spectrum is allocated, with available bands being limited by a network operator's allocation (Nekouei, Alpcan and Dey, 2012). In addition, regulations relating to interference levels, including low power spectral density limits, restrict the use of spectrum sharing technologies such as ultra-wideband wireless technology, which allows devices to share wide bands of spectrum with other users while appearing to those other users only as extra background noise (Suzuki et al., 2007).

[83]Indeed, as discussed by Hellwig (2003, Section 7), there is a notion that requiring public projects to be self-financed creates just about the right incentives for public servants to select projects appropriately.

[84]Public officials are calling for the auction to deliver substantial revenues. All of the FCC Commissioners have agreed that the auction should be designed to raise at least $7 billion (an amount related to estimates of the cost to develop a nationwide interoperable public safety network), and one has stated that "we should focus on maximizing revenue." A member of Congress has stated that "we want the FCC to design the rules to get us at least $24 billion" and another has interpreted the authorizing legislating as requiring that the auction be designed to maximize revenue. (See footnote 30.)

with little evidence of substantial efficiency losses.[85] However, this experience has been in environments with one-sided private information, which as shown by Theorem 2, allows one to be more aggressive than in two-sided markets in terms of generating revenue without severely sacrificing efficiency. Because the revenue efficiency tradeoff is steeper in two-sided markets, market designers may want to consider ways to mitigate this tradeoff, for example by adding supply.

A concern for policymaking related to two-sided markets is that knowledge and experience based on the design of one-sided markets does not necessarily extend to the design of two-sided markets. Although we have shown that the existing literature provides valuable foundations for the design of two-sided markets, we have also highlighted a number of issues related to such markets that are not well understood and likely cannot be understood simply by analogy to one-sided markets. Key examples include the impact of new possibilities for collusion in two-sided markets (e.g., between buyers and sellers) and the issue of managing complexity in a two-sided market, where uncertainty about supply limits the ability of the designer to reduce complexity through the creation of prepackaged bundles that account for complementarities and substitutability among the objects being sold. For the time being, some cautious maneuvering in the fog will be inevitable. The synthesis we provide offers discussion that can guide market design efforts and related research going forward.

[85]Total bids have exceeded $50 billion according to FCC Commissioner Jessica Rosenworcel's statements at the Congressional Hearing on "Keeping the New Broadband Spectrum Law on Track" (U.S. House Energy and Commerce Committee, 12 Dec. 2012) despite the FCC's generally conservative approach to setting reserve prices (see footnote 23). Fox and Bajari (2013) provide evidence of inefficiencies in spectrum license allocations related to the geographic coverage of the licenses offered being inefficiently small. See also the references in footnote 24.

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